VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: z transform part 2

Course Title: signal and system

Paper: ELT-A-CC-4-10-TH

Unit: z transform

Semester: fourth

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Name of the Department: Electronic science

(1) Find z transform & ROC for causal sequence.

$$x(n) = \{\underset{\uparrow}{\overset{x(0)}{1}}, \overset{x(1)}{0}, \overset{x(2)}{-3}, \overset{x(3)}{2}, \overset{x(4)}{5}\}$$

$$X_+(Z) = x(0)Z^{-0} + x(1)Z^{-1} + x(2)Z^{-2} + x(3)Z^{-3} + x(4)Z^{-4}$$

$$= 1 + 0 - 3Z^{-2} + 2Z^{-3} + 5Z^{-4}$$

ROC is entire z-plane except $z = 0$

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$$\{\overset{x_{-4}}{-3}, \overset{x_{-3}}{-2}, \overset{x_{-2}}{-1}, \overset{x_{-1}}{3}, \overset{x_0}{2}\} \qquad x(n)z^{-n}$$

$$X_-(Z) = \sum_{-4}^{0} x(-4)Z^{+4} + x(-3)Z^{+3} + x(-2)Z^{+2} + x(-1)Z^{+1} + x(0)Z^{0}$$

$$= -3Z^{+4} - 2Z^{+3} - 1Z^{+2} + 3Z^{+1} + 2$$

ROC is entire z-plane except $z = \infty$.

property :-

linearity $\rightarrow Z[a x_1(n) + b x_2(n)]$

$$= a X_1(Z) + b X_2(Z)$$

Time shift = $Z[x(n-m)] = Z^{-m} X(Z)$

$m > 0$

$$Z[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{x(n-m) Z^{-(n-m)}}{Z^{-(n-m)}} Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n-m) Z^{-(n-m)} Z^{-m}$$

$$= Z^{-m} \sum_{l=-\infty}^{\infty} x(l) Z^{-l} = Z^{-m} X(Z)$$

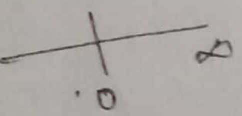
3) If $X_+(z) = Z[x(n)]$

i) $Z[x(n-m)] = z^{-m}\left\{ X_+(z) + \sum_{k=1}^{m} x(-k)\, z^{k} \right\}$

$$Z[x(n-m)] = \sum_{n=0}^{\infty} x(n-m)\, z^{-n}$$

$n = m+1$

$$= z^{-m} \sum_{l=-m}^{\infty} x(l)\, z^{-l}.$$



$\left[\begin{array}{l} n-m = l. \\ n=0,\ l=-m \\ n=\infty \ \ l=\infty \end{array} \right.$

$$= z^{-m}\left\{ \sum_{l=0}^{\infty} x(l)\, z^{-l} + \sum_{l=-m}^{l=-1} x(l)\, z^{-l} \right\}$$

$$= z^{-m}\left\{ X_+(z) + \sum_{k=1}^{m} x(-k)\, z^{k} \right\}.$$

$\left[\begin{array}{l} l = -k. \\ +m = +k \end{array} \right.$

(4) multiply by an exponential sequence.

$X(z) = x(n) z^{-n}$

$$Z[a^n x(n)] = X(a^{-1}z).$$

proof.

$$Z[a^n x(n)] = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) (a^{-1}z)^{-n}$$

$$= \sum x(a^{-1}z).$$

(5) Time reversal

$$Z[x(-n)] = X(z^{-1})$$

proof

$$Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

$$= \sum_{l=\infty}^{-\infty} x(l) z^{l}.$$

$$= \sum x(l) (z^{-1})^{-l}.$$

$$= X(z^{-1}).$$

differentiation

$$Z[n x(n)] = -z \frac{d}{dz} x(z).$$

proof

$$\cancel{Z[n x(n)] = \sum_{n=-\infty}^{\infty} n x(n) z^{-n}}.$$

$$\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dz} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \{-n (z^{-n-1})\}$$

$$= -\frac{1}{z} \sum_{n=-\infty}^{\infty} n \{ x(n) z^{-n}.$$

$$-z \frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} n x(n) z^{-n}$$

$$= Z(n x(n))$$