VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED 'A' GRADE

Topic: z transform

Course Title: signal and system

Paper: ELT-A-CC-4-10-TH

Unit: z transform

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Name of the Department: Electronic science

Z transform

Z transform of a discrete time sequence $x(n)$ is
$$X(z) = Z[x(n)]$$

$$z = re^{j\omega}$$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

$$X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)(re^{j\omega})^{-n}, \quad r = 1$$

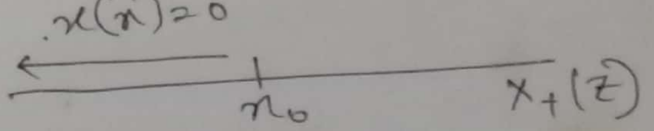
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

Z- transform for a finite duration sequence

Since the z- transform is an infinite power series with $n = -\infty$ to ∞. it exist only for those values of z for which the series converges. Therefor each fun will have a region of convergence (ROC) of $X(z)$ which is the set of all values of z for which $X(z)$ atains a finite value.

Right handed sequence

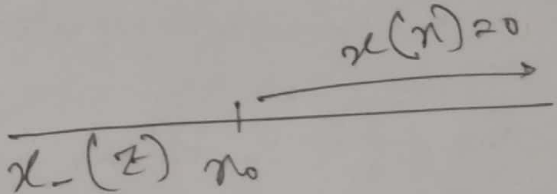
A right handed sequence for which $x(n) = 0$ for all $n < n_0$. number can be a +ve or -ve but finite.



← signal exists only at right hand part.

if number is greater then or equal to '0' the resulting sequence is called casual or positive time sequence

Left handed sequence is one for which $x_n = 0$ for all $n \geq n_0$ when $n_0 \rightarrow +ve$ -ve of finite. if no. is less then or equal to '0' the resulting sequence or ~~neg~~ negative time sequence.



Casual

$$X_+(Z) = \sum_{n=0}^{\infty} x(n) Z^{-n}$$

ROC is in all Z-plane except $Z=0$

anticasual

$$X_-(Z) = \sum_{n=\infty}^{-1} x(n) Z^{-n}$$

ROC is in all Z-plane except $Z=\infty$.

1) for casual sequence —

$$x(n) = a^n u(n)$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n Z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$$

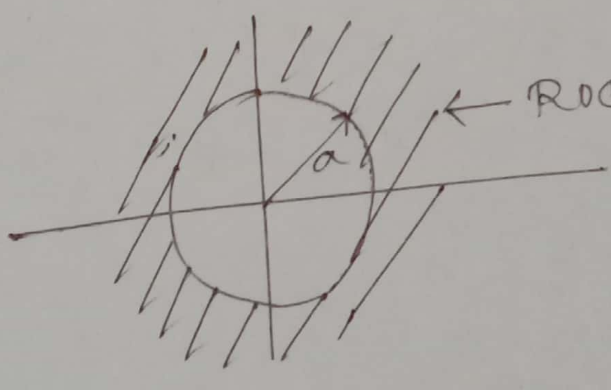
from G.P series.

$$r^n = 1 + r + r^2 + r^3 + \dots = \frac{1}{1-r}, \quad |r| < 1$$

$$X(Z) = \frac{1}{1 - aZ^{-1}}$$

$$ROC = |aZ^{-1}| < 1$$

$$\text{or, } |a| < |Z|$$



2) for anticasual

$$x(n) = -b^n u(-n-1)$$

$$u(-n-1) = 0 \quad n \geq 0$$

$$= 1 \quad n \leq 1$$

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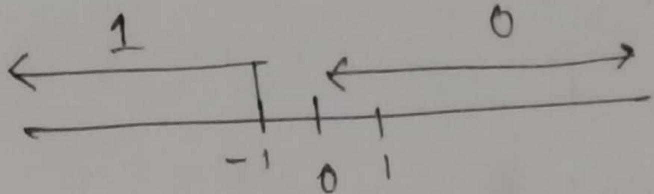


$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} b^{n} z^{-n}$$

$$= -\sum_{n=1}^{\infty} b^{-n} z^{n}$$

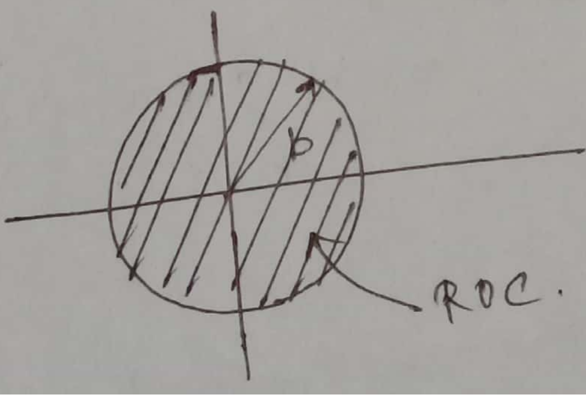
$$= -\sum_{n=0}^{\infty}\left[\left(b^{-n} z^{n}\right) - 1\right] = -\sum_{n=0}^{\infty}\left[\left(b^{-1} z\right)^{n} - 1\right]$$

$$= -\left[\frac{1}{1-b^{-1}z} - 1\right] = -\left[\frac{b}{b-z} - 1\right]$$

$$= \left[\frac{z}{z-b}\right]$$

ROC $|b^{-1} z| < 1$

$|z| < |b|$



ROC.

Stability & ROC

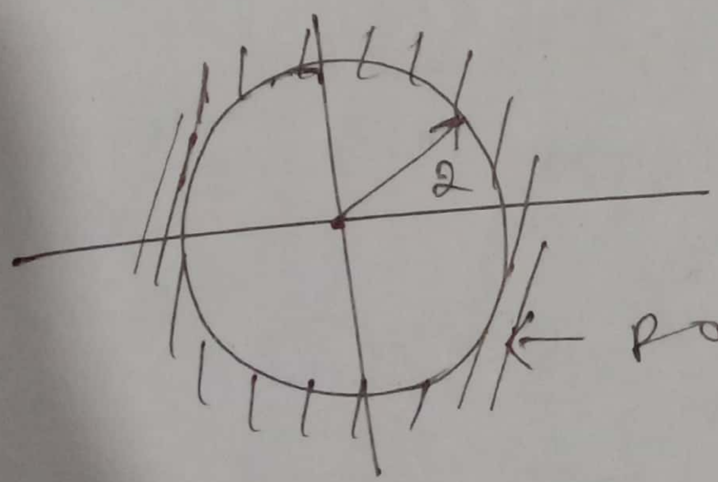
$x(n)$ be the impulse response of casual & anticasual LTI system with the Z transform as $H(Z)$ then the stability of system can be determined from ROC using following theorem.

Theorem

A LTI system with $H(z)$ has the system fun^n is stable if & only if the ROC of $H(z)$ contains the unit circle.

$h(n) = 2^n u(n) \qquad H(z) = \frac{z}{z-2}$

$|z| > 2$



ROC so system is unstable