VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED 'A' GRADE

Topic: Boolean Algebra

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Boolean Algebra and Minimization Techniques

Boolean algebra differs from conventional algebra. This algebra deals with the rules by which the logical operations are carried out.

Boolean logic operations :-

Basic logic function include the OR function, the ~~And~~ AND function and NOT function.

(i) Logical AND operation :-

This operation is logical multiplication operation.

The logical AND operation of two Boolean variables A and B. so output $Y = A \cdot B$ The truth tables shows, the result of the AND operation on the variables A and B is logical 0 for all cases, except when both A and B are logical 1.

Inputs		outputs (Y)
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

ii) Logical OR operation :-

The OR operation between two variables A and B is $Y = A + B$

The symbol use for this operation is '+'.

The truth table shows that when A or B or both are '1' then result is '1'

Inputs		Output
A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

iii) NOT or Logical complementation

In this case, logic '1' is inverted to '0' and logic '0' inverted to '1'

and complement of a variable say, A is $(\overline{A})$.

Basic Laws of Boolean Algebra :-

① Boolean addition :-

This is same as OR operation

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

② Boolean Multiplication :-

This method is same as logical AND operation.

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

Properties of Boolean Algebra

① Commutative property :-

In this property, operation conducted on the variables makes no difference.

i) Boolean addition is commutative

$$A + B = B + A$$

ii) AND operation

$$A \cdot B = B \cdot A$$

Associative property :-

In this law, it makes no difference in what order the variables are grouped

i) For OR operation

$$A + (B + C) = (A + B) + C$$

ii) FOR AND operation

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Distributive property :-

This property states that AND operation of several variables and then the OR operation of the result with a single variable is equivalent to OR operation of single variable with each of the several variables and then the AND operation of the sums.

$$A + BC = (A + B)(A + C)$$

proof

$$\begin{aligned} A + BC &= A \cdot 1 + BC \\ &= A(1 + B) + BC \quad [\because (1+B) = 1] \\ &= A \cdot 1 + AB + BC \\ &= A \cdot (1 + C) + AB + BC \\ &= A \cdot 1 + AC + AB + BC \\ &= A \cdot A + AC + AB + BC \quad (\because A \cdot A = A) \\ &= A(A + C) + B(A + C) \\ &= (A + B)(A + C) \quad [\text{proved}] \end{aligned}$$

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iii) $A \cdot (B+C) = A \cdot B + A \cdot C$

according to this property, OR operation of several variables and then the AND operation of the result with a single variable is equivalent to AND operation of single variable with each of several variables and then the OR operation of the products.

Absorption laws :-

i) $A + AB = A$

$A + AB$

$\Rightarrow A(1+B)$

$\Rightarrow A \quad [\because (1+B) = 1]$.

ii) $A \cdot (A+B) = A$

iii) $A + \bar{A}B = A + B$.

$A + \bar{A}B$

$\Rightarrow (A+\bar{A}) \cdot (A+B)$ [from distributive property]

$\downarrow$

$\Rightarrow 1 \cdot (A+B)$.

Boolean Laws :-

1) $A + 0 = A$

2) $A + 1 = 1$

3) $A + A = A$

4) $A \cdot 1 = A$

5) $A \cdot 0 = 0$

6) $A \cdot A = A$

7) $A + \bar{A} = 1$

8) $A \cdot \bar{A} = 0$

9) $\bar{\bar{A}} = A$

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DEMORGAN'S THEOREMS :-

1) first theorem states that the complement of a product is equal to the sum of the complemen

$$\overline{AB} = \overline{A} + \overline{B}$$

2) The second theorem states that, the complement of sum is equal to the product of the complements.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

Proof of DeMorgan's theorem by perfect induction method :-

(1) $A = 0 \quad \overline{A} = 1$
$B = 0. \quad \overline{B} = 1$

so from first theorem,

$$\overline{AB} = \overline{0.0} = \overline{0} = 1$$
$$\overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 \quad \text{Proved}$$

for second theorem,

$$\overline{A+B} = \overline{0+0} = \overline{0} = 1$$
$$\overline{A} \cdot \overline{B} = 1 \cdot 1 = 1$$
$$\text{so, } \overline{A+B} = \overline{A} \cdot \overline{B}$$

(2) $A = 0 \quad \overline{A} = 1$
$B = 1 \quad \overline{B} = 0$

for 1st theorem

$$\overline{AB} = \overline{0 \cdot 1} = \overline{0} = 1$$

$$\overline{A} + \overline{B} = 1 + 0 = 1$$

$$\text{so, } \overline{AB} = \overline{A} + \overline{B}$$

for 2nd Theorem

$$\overline{A+B} = \overline{0+1} = \overline{1} = 0$$

$$\overline{A} \cdot \overline{B} = 1 \cdot 0 = 0$$

$$\text{so } \overline{A+B} = \overline{A} \cdot \overline{B}$$

(3) $A = 1 \quad \overline{A} = 0$
$B = 0 \quad \overline{B} = 1$

for, 1st theorem,

$$\overline{AB} = \overline{1 \cdot 0} = \overline{0} = 1$$

$\overline{A} + \overline{B} = 0 + 1 = 1$ proved

for, 2nd theorem,

$$\overline{A+B} = \overline{1+0} = \overline{1} = 0$$

$\overline{A} \cdot \overline{B} = 0 \cdot 1 = 0$ proved.

(4) $A = 1 \quad \overline{A} = 0$
$B = 1 \quad \overline{B} = 0$

for, 1st theorem,

$$\overline{AB} = \overline{1 \cdot 1} = \overline{1} = 0$$

$\overline{A} + \overline{B} = 0 \cdot 0 = 0$ proved.

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for 2nd theorem,

$$\overline{A+B} = \overline{1+1} = \overline{1} = 0$$

$$\overline{A}\cdot\overline{B} = 0.0 = 0$$

Also we can prove by algebraic method,

① First theorem, $\overline{AB} = \overline{A}+\overline{B}$

so, according to this, $\overline{A}+\overline{B}$ is complement of AB. So if this true then we can say from complementarity law $X+\overline{X}=1$ and $X\overline{X}=0$ is true.

so, $X = AB$ and complement of X, i.e

$\overline{X} = \overline{A}+\overline{B}$

$$X+\overline{X} = 1$$

$\Rightarrow AB + (\overline{A}+\overline{B}) = 1$

$\Rightarrow (\overline{A}+\overline{B}+A)\cdot(\overline{A}+\overline{B}+B) = 1$ [from distributive law].

$\Rightarrow (1+\overline{B})\cdot(1+\overline{A}) = 1$

$\Rightarrow 1 = 1$ [proved]

$$X\overline{X} = 0$$

$\Rightarrow AB(\overline{A}+\overline{B}) = 0$

$\Rightarrow AB\overline{A} + AB\overline{B} = 0$

$\Rightarrow 0+0 = 0$ [proved].

② For second theorem, $\overline{A+B} = \overline{A}\cdot\overline{B}$

so, $\overline{A}\cdot\overline{B}$ is complement of A+B.

so, $\left.\begin{array}{l} X = A+B \\ \overline{X} = \overline{A}\cdot\overline{B} \end{array}\right\}$ let.

$$X+\overline{X} = 1$$

$\Rightarrow (A+B)+\overline{A}\cdot\overline{B} = 1$

$\Rightarrow \{(A+B)+\overline{A}\}\cdot\{(A+B)+\overline{B}\} = 1$

$\Rightarrow \{(1+B)\}\{(1+A)\} = 1$

$\Rightarrow 1\cdot 1 = 1$ proved

from distributive law, $X+YZ = (X+Y)(X+Z)$

where, $X = A+B$

$Y = \overline{A}$

$Z = \overline{B}$.

$$X \cdot \overline{X} = 0$$

$$(A + B) \cdot (\overline{A} \cdot \overline{B}) = 0$$

$$(A \cdot \overline{A}\ \overline{B}) + (B \cdot \overline{A} \cdot \overline{B}) = 0$$

$$0 + 0 = 0$$ Proved.