VIVEKANANDA COLLEGE

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KOLKATA-700063

NAAC ACCREDITED 'A' GRADE

Topic: signals and systems

Course Title: Signals and Systems

Paper: CC-10

Unit:

Semester: 4

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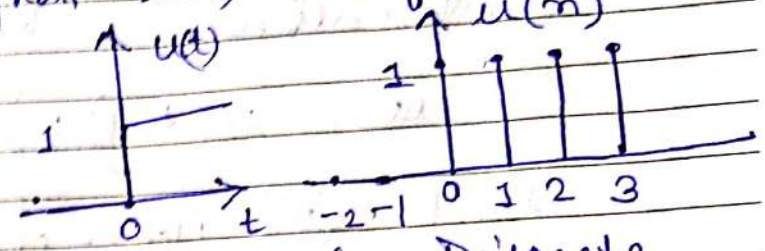
Name of the Department: Electronics

Signals and Systems

Unit Step Signal (Continuous domain u(t), discrete domain u[n])

The unit step signal u(t) is defined as

$$u(t) = \begin{cases} 1, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$$



(a) Continuous domain (b) Discrete domain

The unit step sequence u(n) is defined as

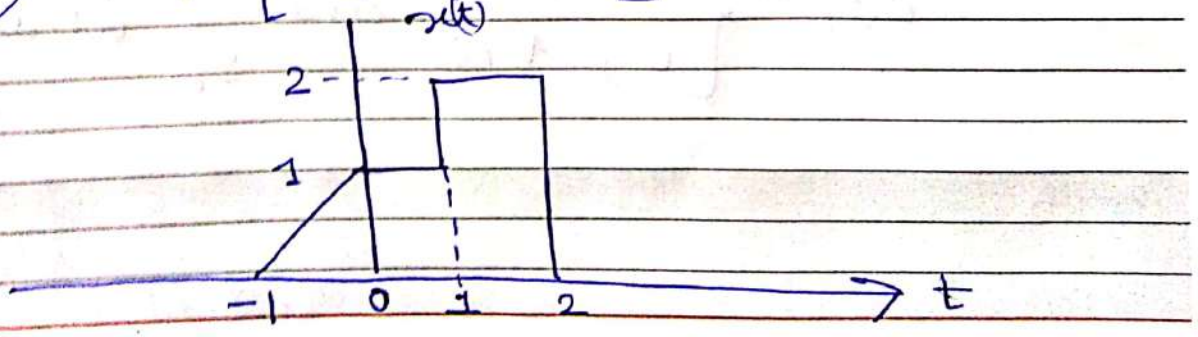
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

The value of u[n] at n=0 is defined. [unlike the continuous time step function u(t) at t=0] and equals to unity.

EX: A continuous time signal x(t) is shown in fig. Sketch and label each of the following signals.

(a) x(t) u(t-1)

(b) x(t) [u(t) - u(t-1)]



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Solution

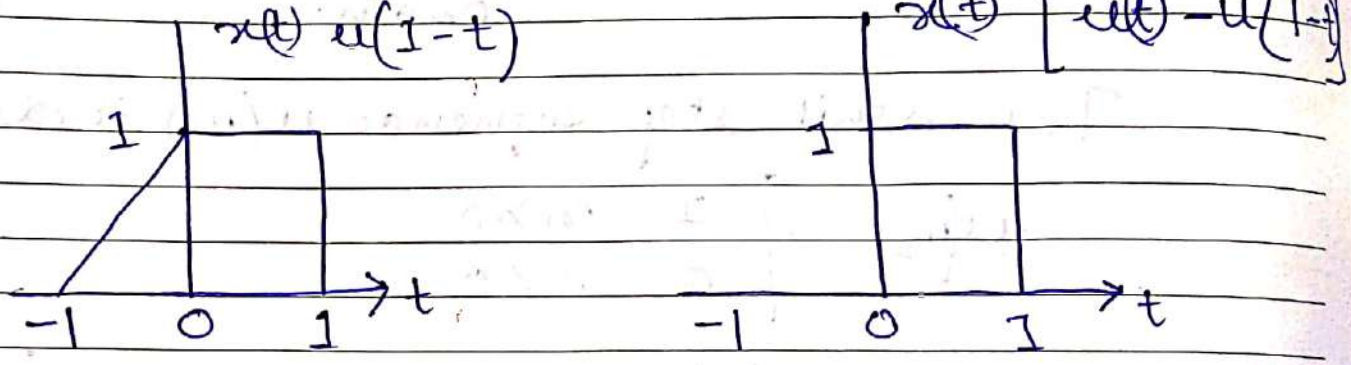
(a) By definition

$u(1-t) = \begin{cases} 1 & t<1 \\ 0 & t>1 \end{cases}$ and $x(t)u(1-t)$ is sketched in ex fig.

(b) By the definition of unit step function combination,

$u(t) - u(t-1) = \begin{cases} 1 & 0<t<1 \\ 0 & \text{otherwise} \end{cases}$

and $x(t)[u(t)-u(t-1)]$ is sketch in below fig

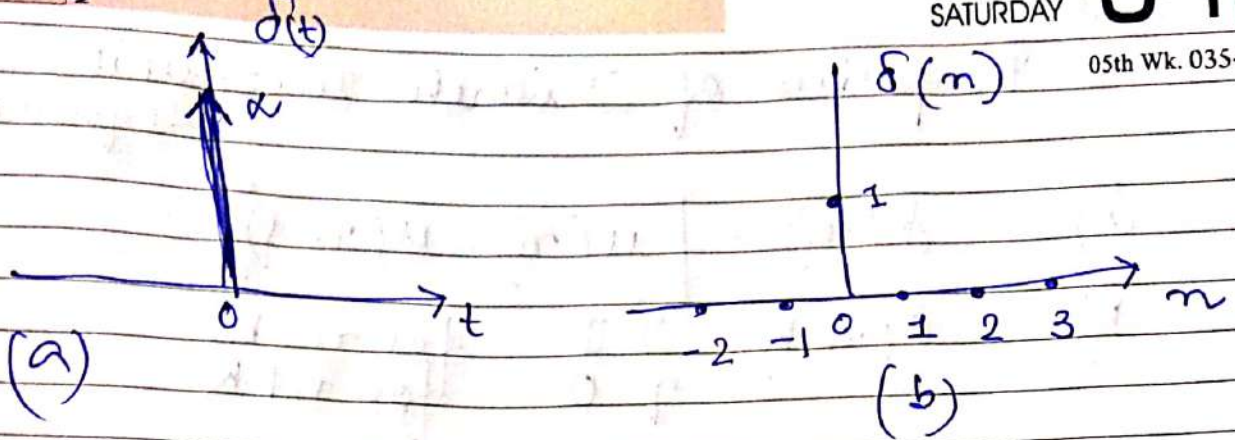


→ Unit Impulse Signal (Continuous Domain $\delta(t)$, Discrete Domain $\delta(n)$)

The impulse signal is a signal with infinite magnitude and zero duration, with unit area Mathematically, impulse signal is defined as

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

Remember



Unit impulse signal (a) Continuous domain
(b) Discrete domain.

The unit impulse (or unit sample) sequence $\delta(n)$ is defined as

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Properties of continuous-time unit impulse function

(a) It is an even function of time t, ie $\delta(t) = \delta(-t)$

(b) $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

(c) $\delta(at) = \frac{1}{|a|}\delta(t)$

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(d) $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$; $x(t)\,\delta(t) = x(0)\,\delta(t)$

(e) $\int_{-\infty}^{\infty} \delta(t_0)\,x(t)\,dt = -x(0)$

(f) $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$ or

$x(t) = x(t) * \delta(t)$

Properties of Discrete time unit sample sequence

a) $\delta(n) = [u(n) - u(n-1)]$

b) $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$

c) $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

d) $x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$

e) $\delta(kn) = \delta(n)$

→ Relation between Step and impulse signal

Discrete Time Domain	Continuous Time Domain
$\delta(n) = [u(n) - u(n-1)]$	$\delta(t) = \frac{du(t)}{dt}$
$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$	$u(t) = \int_0^{\infty} \delta(t-\tau)\,d\tau$
$u[n] = \sum_{m=-\infty}^{n} \delta[m]$	$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

Ex. Evaluate the following integral.

(a) $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt$

Sol^n By eq., $\int_a^b \phi(t)\,\delta(t-\tau)\,dt = \begin{cases} \phi(\tau) & a<\tau<b \\ 0 & a<b<\tau \\ \text{undefined} & a=\tau \text{ or } b=\tau \end{cases}$

with $a=-1$, $b=1$ & $\tau=0$;

$\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt = |(3t^2+1)|_{t=0} = 1$