VIVEKANANDA COLLEGE

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NAAC ACCREDITED 'A' GRADE

Topic: signals and systems

Course Title: Signals and Systems

Paper: CC-10

Unit:

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Name of the Department: Electronics

Signals & systems

CC-10

Convolution sum

Following are the convolution integral properties

a) Commutative

$$x(n) * h(n) = h(n) * x(n)$$

b) Associative

$$\{x(n) * h_1(n)\} * h_2(n) = x(n) * \{h_1(n) * h_2$$

Two cascade connected discrete time systems with impulse response $h_1[n]$ and $h_2[n]$ can be replaced by a single equivalent discrete time system whose impulse response is given by convolution of individual impulse responses.

~~c) Associative~~

~~$\{x(n) * h_1(n)\} * h_2(n) = x(n) * \{h_1(n) * h_2($~~

c) Distributive

Two parallel connected discrete time systems with impulse responses $h_1[n]$ and $h_2[n]$ can be replaced by a single equivalent discrete time system whose impulse response is given by the sum of individual impulse responses.

d) Convolution with an impulse

Convolution of a signal $x(n)$ with an unit impulse function results in the signal $x(n)$ itself

$$x(n) * \delta(n) = x(n)$$

e) Shifting property

if $x(t) * h(t) = y(t)$, then $x(t) * h(t - t_0) = x(t - t_0) * h(t) = y(t - t_0)$

and $x(t - t_1) * h(t - t_2) = y(t - t_1 - t_2)$

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f) Differentiation property

If $x(t) * h(t) = y(t)$, then $\left[\frac{d}{dx} x(t)\right] * h(t) = x(t) * \left[\frac{d}{dt} h(t)\right] = \frac{d}{dt} y(t)$

Remember

This property states that if both $x(t)$ & $h(t)$ are time scaled by a, their convolution is also time scaled by a and multiplied by $\frac{1}{|a|}$.

f) Time scaling property

If $x(t) * h(t) = y(t)$, then $x(at) * h(at) = \frac{1}{|a|} y(at)$

This property states that if both $x(t)$ and $h(t)$ are time scaled by a, their convolution is also time scaled by a and multiplied by $1/|a|$.

Ex: A signal $x(t)$ has energy E. Calculate the energy of the signal $x(3t)$

Soln

The energy of the signal $x(t)$ is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\therefore E' = \int_{-\infty}^{\infty} |x(3t)|^2 \, dt$$

Let $3t = p$

$$\therefore dt = \frac{dp}{3}$$

$$\therefore E' = \int_{-\infty}^{\infty} x(p)^2 \frac{dp}{3} = \frac{1}{3} \int_{-\infty}^{\infty} |x(p)|^2 \, dp$$

$$= \frac{1}{3} E$$

Remember

Home work

Ex.1 $x(t) * u(t) = ?$

Ex 2 A linear time invariant system has an impulse response e^{2t}, $t>0$. If the initial conditions are zero and input is e^{3t}, the o/p for $t>0$ is

a) $e^{3t} - e^{2t}$ b) e^{5t}

c) $e^{3t} + e^{2t}$ D) None

Causal and Non causal system

A system is called causal if its o/p $y(t)$ at an arbitrary time $t = t_0$ depends only on the input $x(t)$ for $t \leq t_0$. That is, the o/p of a causal system at the present time depends on only the present and past values of the input, not on its future values. Thus in a causal system, it is not possible to obtain an output before an input is applied to the system. A system is called non-causal if its o/p depends on future values of the input along with present and past input values. Ex examples of non causal

$y(t) = x(t+1)$

$y(n) = x(-n)$

Ex → The discrete LTI systems with the following impulse response is non causal.

(a) $a^n u(n-2)$ (b) $a^{n-2} u(n)$ (c) $a^{n+2} u(n)$

(d) $a^n u(n+2)$

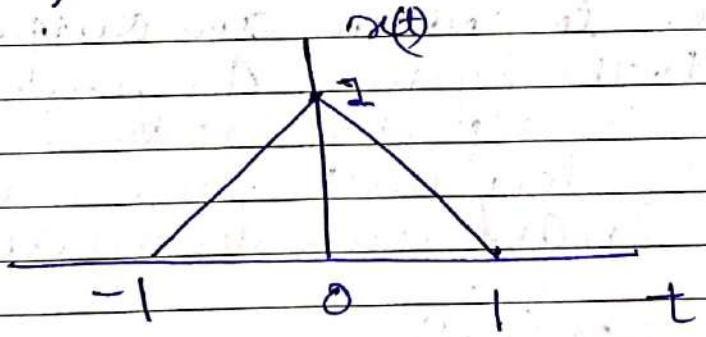
Soln — A discrete time LTI system is said to be non causal if its impulse response is non zero for negative time.

Thus signal $a^n u(n+2)$ is non causal.

Ex → A triangular pulse signal $x(t)$ is depicted in fig. Sketch each of the following signals derived from $x(t)$.

(a) $x(3t)$ b) $x(3t+2)$

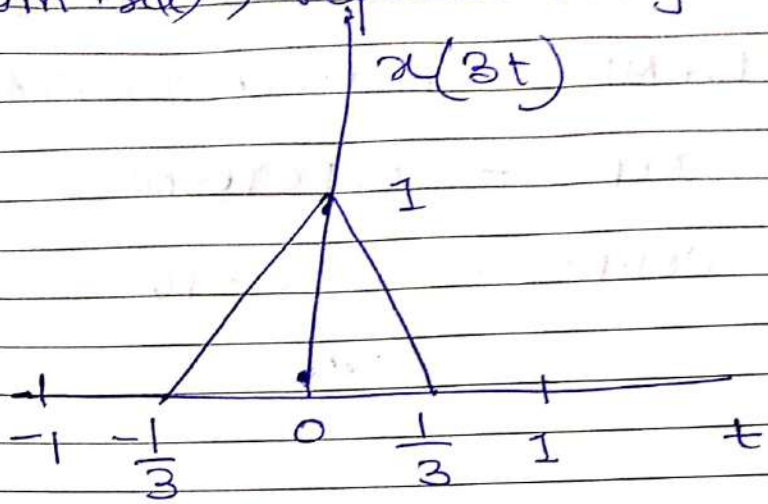
(c) $x(-2t-1)$



Soln — The mathematical form of $x(t)$ is

$$x(t) = \begin{cases} 1+t ; & -1 \le t \le 0 \\ 1-t ; & 0 \le t \le 1 \end{cases}$$

(a) To obtain the mathematical form of $x(3t)$ from $x(t)$, replace t by $3t$ in $x(t)$ as



$$x(3t) = \begin{cases} 1+3t; & -1 \le 3t \le 0 \rightarrow -\frac{1}{3} \le 0 \\ 1-3t; & 0 \le 3t \le 1 \rightarrow 0 \le t \le \frac{1}{3} \end{cases}$$

H/W Do b) and c) using the above process.