STUDY MATERIAL

VIVEKANANDA COLLEGE, THAKURPUKUR

NAAC ACCREDITED GRADE—'A'

ZOOLOGY

(HONOURS)

LOTKA-VOLTERRA PREDATION MODEL

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LOTKA-VOLTERRA PREDATION MODEL

About Predation: In restricted sense predation means members of one species (predator) eating those of other species (prey) and thus involves killing of the prey. But predation in its broadest sense may be defined as one organism feeding on another living organism and thus include herbivory, carnivory, cannibalism and parasitoid. It is the negative inter-specific association where one predator kills and feed on the prey. It is a step in transfer of energy.

Application of Mathematics in Ecology by Lotka and Volterra: An application of the nonlinear system of differential equations in mathematical biology / ecology is to model the predator-prey relationship of a simple ecosystem. Suppose in a closed ecosystem (i.e. no migration is allowed into or out of the system) there are only two types of animals: the predator and the prey. They form a simple food chain where the predator species hunts the prey species, while the prey grazes vegetation. The size of the two populations can be described by a simple system of two nonlinear first order differential equations (the Lotka-Volterra equations).

Two mathematicians, Lotka (1925) and Volterra (1926) independently developed mathematical model to express the relationship between predator and prey. The model involves two equations one for the predator and another for the prey.

Supposition: The equations are based on two suppositions:-

1. Number of prey will increase in the absence of predator and
2. Number of predator will decrease in the absence of prey.

Equations:

In an unlimited environment, populations show the exponential growth which may be expressed as:

$$dN / dt = r N$$

[N= population size, t= time, r = per capita rate of population growth, dN = rate of change in number, dt = rate of change in time, dN/dt = rate of population increase]

In the absence of predator (P) the prey population (N) is expected to increase:

$$dN / dt = r_1 N \text{------------------------} 1$$

[N= prey density and r_1= intrinsic rate of increase of prey population in the absence of predator]

Similarly the predator population is expected to decrease in the absence of prey:

$$dP / dt = - r_2 P \text{-------------------------} 2$$

[P= density of predator and $– r_2$= instantaneous specific death rate of predator in the absence of prey]

If the predator and prey are put together in the limited space the rate of increase of prey will be slowed down by a factor that depends on the density of predator:

$$dN / dt = (r_1 - \varepsilon P)N \text{------------------} 3$$

[ε is a constant measuring ability of prey to escape predator]

Thus, ‘$-\varepsilon P$’ denotes decrease in the prey density per individual of predator present which is a negative phenomena.

Similarly, in such situation, the predator population will increase at a rate that depends on the density of the prey population:

$$dP / dt = (-r_2 + \theta N)P \text{-------------------} 4$$

[θ is a measure of skill of the predator in catching prey]

Thus, 'θN' is the increase in the predator population per individual of prey present.

The Lotka-Voltera model shows that as the predator population increases the prey population decreases to a point which reversed the influence. They prey neither quite destroyed and the predator never completely exterminate. Thus both the population oscillates through time (Fig. 1).

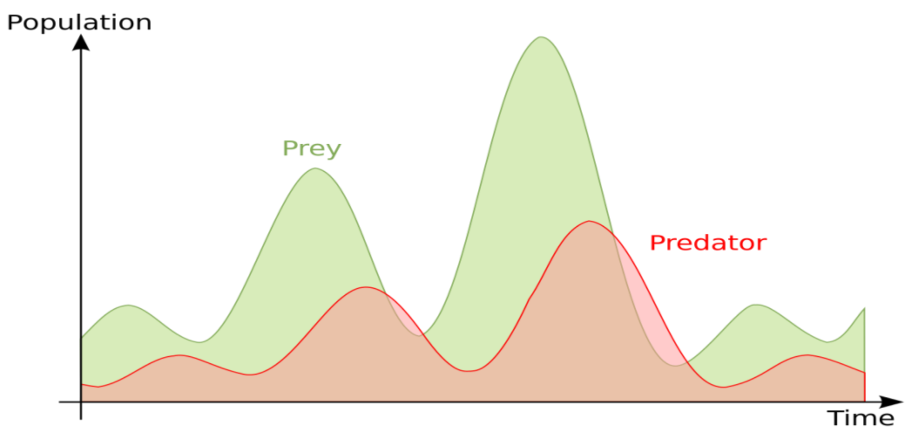

Fig.1-Classical predator-prey cycle predicted by Lotka and Volterra.

Experimental Evidence by G. F. Gause, 1934:-

A. **First sets of experiment:** Gause introduced five *Paramecium caudatum* (Prey, Fig. 2) in a small test tube containing oat media which promote the growth of bacteria that are the food resources of prey. Two days later three *Didinium nasutum* (Predator, Fig. 3) were introduced. Three and four days later, respectively, both the *Paramecium* and the *Didinium* were extinct.

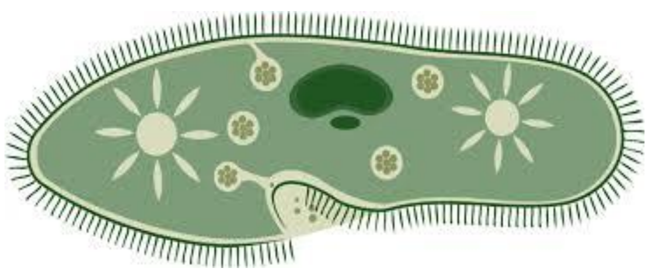

Fig.2- *Paramecium caudatum* (the prey).

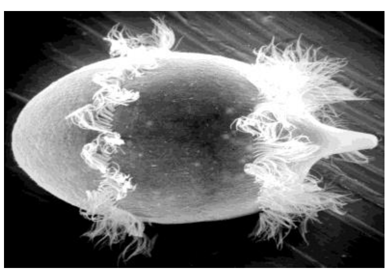

Fig.3- *Didinium nasutum* (the predator).

It was seen that the increase and decrease of prey was a phase ahead of the successive increase and decrease of the predator (Fig. 4).

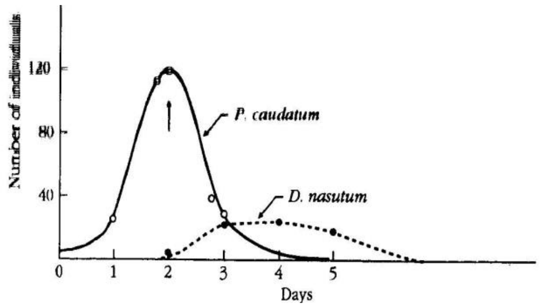

Fig.4-Predator-prey relationship in homogeneous environment

B. **Second sets of experiment:** In the second sets of experiment Gause then modified the homogeneous oat medium environment by introducing some sediment that, in effect, created what he called 'refuges' where paramecia could hide and avoid predator. Now predators and prey were introduced simultaneously. It was seen that population of prey continued to increase and that of predator because extinct (Fig.-5). From this study he concluded that the importance of heterogeneity of the medium and significant role played by it in population interaction.

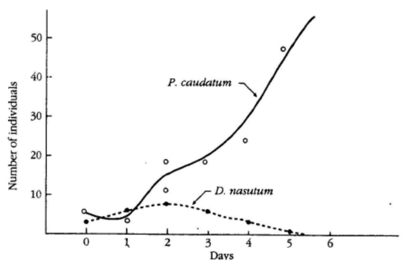

Fig.5- Predator-prey relationship in heterogeneous environment

C. **Third sets of experiment:** In the third sets of experiment Gause was able to show the oscillatory pattern of predator and prey relationship. In these sets of experiments in homogeneous medium, one *Paramecium* and one *Didinium* were introduced on the first day and every three days thereafter. In this manner by periodic introduction of prey to the medium Gause was able to maintain predator and prey populations together and to produce regular fluctuations in both populations as predicted by Lotka and Volterra (Fig. 6).

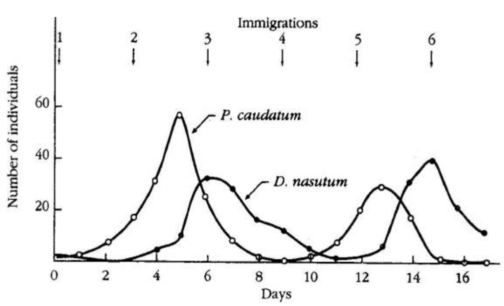

Fig.6- Predator-prey relationship with repeated immigration of both predator and prey.