Study Material
Economics

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Contents

1 Course Information

Semester: IV
Curriculum covered: CC VIII
Course title: Intermediate Microeconomics II
Unit: 1.4

2 Non-Collusive Oligopoly

We will learn the limiting case of oligopoly, viz. duopoly.

2.1 *Cournot* duopoly model

This is the earliest model developed in 1838. The assumptions of this model are:

1. Duopolists have identical products and identical costs. Cournot illustrated his model with the example of two firms each owning a spring of mineral water which is produced at zero costs.

2. They sell their output in a market with a straight-line demand curve.

3. Each firm acts on the assumption that its competitor will not change its output and decides its own output so as to maximise profit.

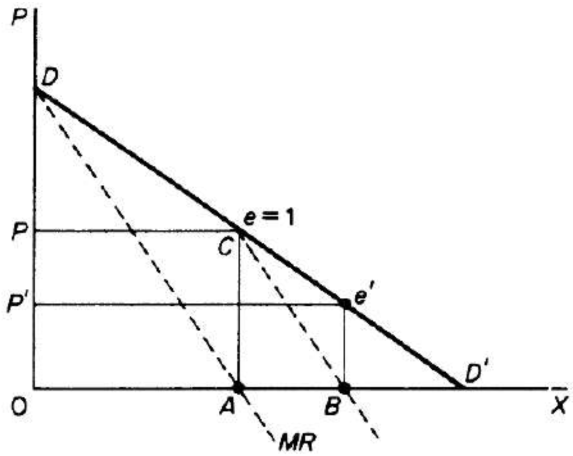


Figure 1

Assume that Firm-A is the first to start producing and selling mineral water. It will produce quantity *A*, at price *P* where profits are at maximum (Figure 1) because *MC*=*MR*=0. The elasticity of market demand at this level of output is equal to unity and the total revenue of the firm is maximum. At zero cost, maximum revenue implies maximum profit. Now, Firm-B assumes that Firm-A will keep its output fixed at *OA* and hence, considers that its own demand curve is *CD'*. Then Firm-B will produce half the quantity *AD'*, viz. *AB* of output and sell at price, *P'* where its profit is maximum. Firm-B produces half of the market which has not been supplied by Firm-A, that is Firm-Bs output is 1/4 (= 1/2 * 1/2) of the total market.

Firm-A, faced with this situation, assumes that Firm-B will retain a constant quantity in the next period. So, it will produce one-half of the market which is not supplied by Firm-B. Since Firm-B covers one-quarter of the market, Firm-A will, in the next period, produce (1/2 * (1 - 1/4)) = (1/2 * 3/4) = 3/8 of the total market. Firm-B reacts on the *Cournot* assumption, and will produce one-half of the non-supplied part of the market, viz. 1/2 (1 - 3/8) = 5/16. In the third period, Firm-A will continue to assume that Firm-B will not change its quantity, and thus, will produce one-half of the remaining part of the market demand, i.e. 1/2 (1 - 5/16). This action-reaction pattern continues till an equilibrium is reached where each firm produces one-third of the total market. The equilibrium of the *Cournot* firms may be obtained as follows:

1. The product of firm *A* in successive periods is

   period 1: $\frac{1}{2}$
   period 2: $\frac{1}{2}(1-\frac{1}{4}) = \frac{3}{8} = \frac{1}{2} - \frac{1}{8}$
   period 3: $\frac{1}{2}(1-\frac{5}{16}) = \frac{11}{32} = \frac{1}{2} - \frac{1}{8} - \frac{1}{32}$
   period 4: $\frac{1}{2}(1-\frac{42}{128}) = \frac{43}{128} = \frac{1}{2} - \frac{1}{8} - \frac{1}{32} - \frac{1}{128}$

   We observe that the output of *A* declines gradually. We may rewrite this expression as follows

$$\begin{bmatrix}\text{Product of } A\\ \text{in equilibrium}\end{bmatrix} = \frac{1}{2} - \frac{1}{8} - \frac{1}{32} - \frac{1}{128} \cdots$$
$$= \frac{1}{2} - [\frac{1}{8} + \frac{1}{8}\cdot\frac{1}{4} + \frac{1}{8}\cdot(\frac{1}{4})^2 + \frac{1}{8}(\frac{1}{4})^3 + \cdots].$$

   The expression in parentheses is a declining geometric progression with ratio $r = \frac{1}{4}$ Applying the summation formula for an infinite geometric series

$$\int = \frac{a}{1-r}$$

   (where $\int$ = sum, a = first term of series, r = ratio) we obtain

$$\begin{bmatrix}\text{Product of } A\\ \text{in equilibrium}\end{bmatrix} = \frac{1}{2} - \frac{\frac{1}{8}}{1-\frac{1}{4}} = \frac{1}{2} - \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{2} - \frac{4}{24} = \frac{8}{24} = \frac{1}{3}$$

2. The product of firm *B* in successive periods is

   period 2: $\frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$
   period 3: $\frac{1}{2}(1-\frac{3}{8}) = \frac{5}{16} = \frac{1}{4} + \frac{1}{16}$
   period 3: $\frac{1}{2}(1-\frac{11}{32}) = \frac{21}{64} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$
   period 4: $\frac{1}{2}(1-\frac{43}{128}) = \frac{85}{256} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$

   We observe that *B*'s output increases, but at a declining rate. We may write

$$\begin{bmatrix}\text{Product of } B\\ \text{in equilibrium}\end{bmatrix} = \frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}(\frac{1}{4})^2 + \frac{1}{4}(\frac{1}{4})^3 + \cdots$$

   Applying the above expression for the summation of a declining geometric series we find

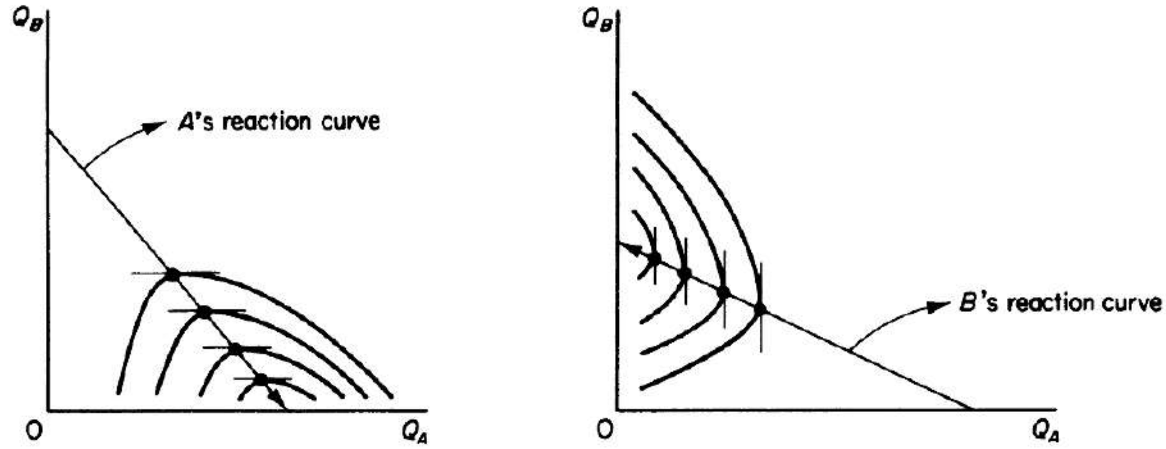
$$\begin{bmatrix}\text{Product of } B\\ \text{in equilibrium}\end{bmatrix} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Thus, the *Cournot* solution is stable. Each firm supplies one-third of the market, at a common price which is zero in this example because of costless production. If there are n firms in the industry, each will provide 1 / (n+1) of the market, and the industry output will be n / (n+1) = 1 / (n+1)*n. *Cournot* model is criticised on the assumption of costless production. However, it can be relaxed without impairing the validity of the model, based on the *Reaction Curves* approach.

2.2 ***Reaction Curves*** approach

The *Reaction Curves* approach is a more powerful method to analyse oligopolistic market because it doesn't require the assumptions of identical costs, costless production and identical demands. This approach is based on the *Stackelberg's indifference curve analysis* which introduces the concept of *isoprofit curves* for competing producers.

An *isoprofit* curve for Firm-A is a string of points showing different levels of output by Firm-A and its rival Firm-B which yields the same level of profit to Firm-A (see Figure 2a). Similarly, an *isoprofit* curve for Firm-B is a string of the points showing different levels of output by Firm-B and its rival which yields the same level of profit to Firm-B (see Figure 2b).



(a) Isoprofit graph for Firm-A (b) Isoprofit graph for Firm-B

Figure 2: Isoprofit maps

The properties of *isoprofit curves* (IP) are:

1. IPs for firms are concave to the axes along which we measure the output of corresponding firm. The Figure 3 shows an *isoprofit* curve of Firm-A which is concave to the horizontal axis Q_A. This shape shows how Firm-A can react to Firm-B's output decisions so as to retain a given level of profit.

   Suppose that Firm-B decides to produce the level of output B_1. A line parallel to the horizontal axis through B1 intersects the *isoprofit* curve Π_{A1} at points *h*

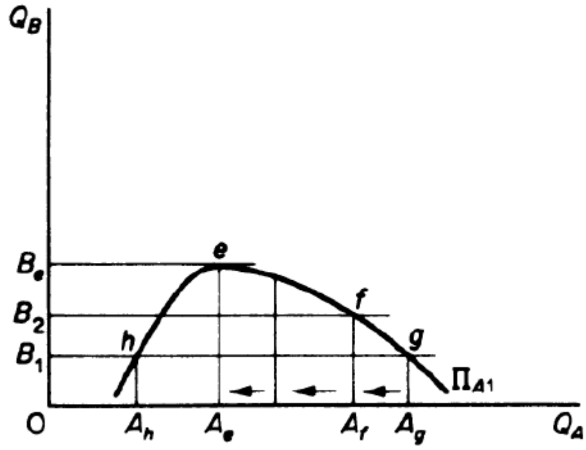


Figure 3

and g. This shows that Firm-A will earn same profit by producing either A_h or A_g corresponding to points h and g. Assume that Firm-A reacts by producing higher production A_g. If now Firm-B increases its output to B_2, Firm-A will reduce its output, otherwise excess production will reduce market price and hence, the profit of Firm-A. Thus, Firm-A will reduce its output in response to increase in the output of Firm-B. This will go on up to a certain point e as shown in Figure 3.

As Firm-A reduces its output, its cost will also change but its net profit , Π=R-C, will remain at the same level because of market elasticity or decreasing cost arising from a better utilisation of Firm-A's plants. Consider point h, if Firm-B decides to increase its output from B_1, Firm-A will react by increasing its output as well. Like before, Firm-A's profit will remain the same because of market elasticity and decrease its costs due to better utilisation of its plants.

2. Further the *isoprofit* curve is from the axes, lower is the profit and vice versa. Consider Figure 4, suppose that Firm-B decides to produce B_4. Firm-A can react by increasing or decreasing or keeping its output level at A_e.

   If Firm-A retains its output at a constant while Firm-B increases its output, market price will fall. As a result, both revenue and profit of Firm-A will decrease if cost of production does not change. If Firm-A increases its production beyond A_e (Figure 4) while Firm-B increases its output, both revenue and profit of Firm-A will decrease due to the inelasticity of demand or increasing cost of production. If Firm-A decreases it's production below A_e, its profit will fall because of elasticity of demand or increasing cost of production. Thus, Firm-A will earn lower levels of profit, no matter what it's reaction is, when Firm-B increases its output beyond B_e. Thus, in Figure 4, the *isoprofit* curve Π_{A2} represents lower profit than Π_{A1}.

3. For Firm-A, the highest points of successive *isoprofit curves* lie to the left of each other. Firm-A's *Reaction Curve* is shown in Figure 5a. If we join the highest points of the *isoprofit curves* we obtain Firm-A's *Reaction Curve*. *Reaction Curve* is de-

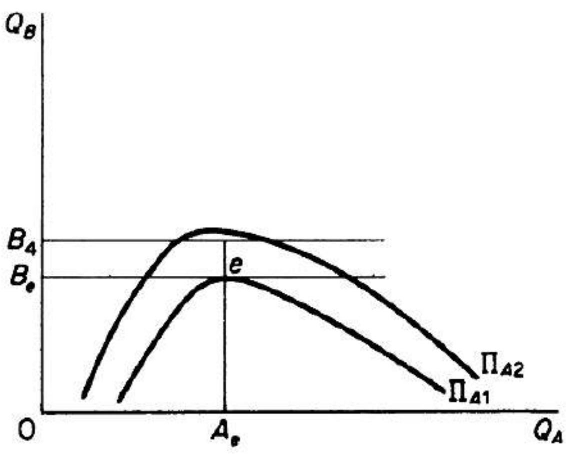
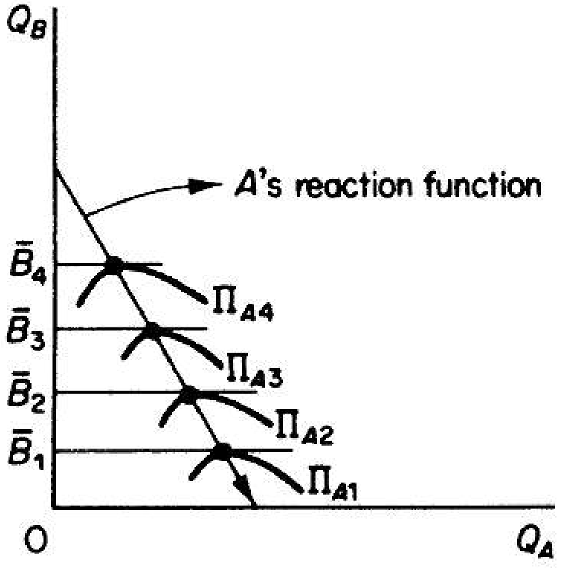
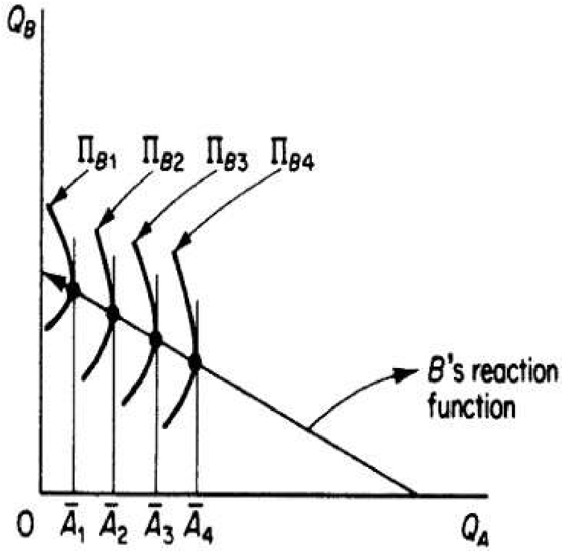


Figure 4

fined as the locus of points representing highest profits that Firm-A can attain, given the different levels of the output of its rival, Firm-B. lt is so called because it shows reaction of Firm-A to Firm-B's decision to increase or decrease production.



(a)



(b)

Figure 5

Firm-B's *isoprofit curves* are concave to the Q_B axis. The highest points of the *isoprofit curves* of B lie to the right of each preceding curves as we move further away from the Q_B axis. If we join these highest points, we obtain Firm-B's *Reaction Curve* (see Figure 5b).

Cournot equilibrium is determined by the intersection of *Reaction Curves* of two firms (see Figure 6). It is a stable equilibrium, provided that Firm-A's *Reaction*

Curve is steeper than Firm-B's *Reaction Curve.*

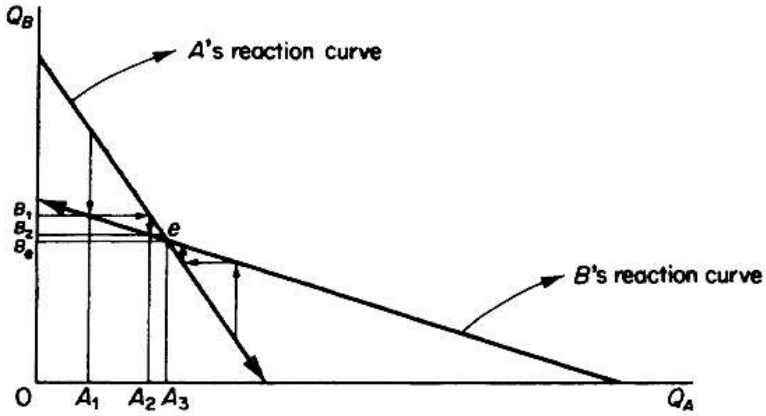


Figure 6

If Firm-A decides to produce A_1, firm B will react by producing B_1 given *Cournot* assumption that Firm-A will keep its quantity fixed at A_1. However, Firm-A reacts by producing higher quantity A_2 on the assumption that Firm-B will stay at the level B_1. Now, Firm-B reacts by producing its output at B_2. This adjustment will continue till the point e is reached. Thus, e is a stable equilibrium but is also suboptimal. At point e each firm maximises its own profit but the joint profit of the industry is not maximised (see Figure 7).

EE' is similar to *Edgeworth's contract curve* which is the locus of the points of tangency of the two firm's *isoprofit curves.* The points on the *contract curve* are optimal because points off this curve imply a lower profit for one or both firms, i.e. less industry profits as compared to points on the curve.

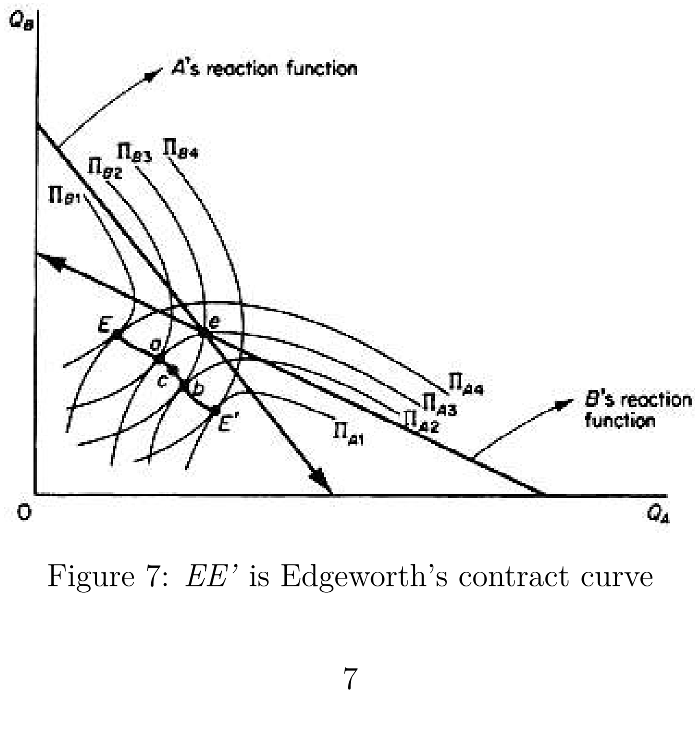


Figure 7: *EE'* is Edgeworth's contract curve

At point a, Firm-A would continue to have same profit Π_{A3} while Firm-B have a higher profit ($\Pi_{B2} > \Pi_{B3}$). Again at any intermediate point between a and b, namely c both Firms will earn higher profits.

QUESTION: Why would competing firms choose a suboptimal equilibrium e?
ANSWER: The *Cournot* pattern of firm's behaviour implies that the firms do not learn from their past experiences, each expecting the other to remain at a given position. *Cournot* duopoly model ends here.

2.3 *Stackelberg* duopoly model

This model is an extension of *Cournot* model and was developed by Heinrich von Stackelberg in 1939. In this model, it is assumed that one duopolist is sufficiently sophisticated to recognise that its competitor acts on as per *Cournot* assumptions. Thus, the wiser duopolist can determine the *Reaction Curve* of his rival and incorporate it in his own profit function, which he then tries to maximise like a monopolist.

Assume that the *isoprofit curves* and the *Reaction* functions of the duopolists are those depicted in Figure 8. If Firm-A is the sophisticated duopolist, it will assume that it's rival will act on the basis of its own *Reaction Curve*. Then it will choose its own output at a level that maximises its own profit. This is point a in Figure 8 which shows the lowest possible *isoprofit* i.e. the highest possible profit Firm-A can earn given Firm-B's*Reaction Curve*. Then, Firm-A acting as a monopolist will produce X_A, and Firm-B will react by producing X_B according to its *Reaction Curve*.

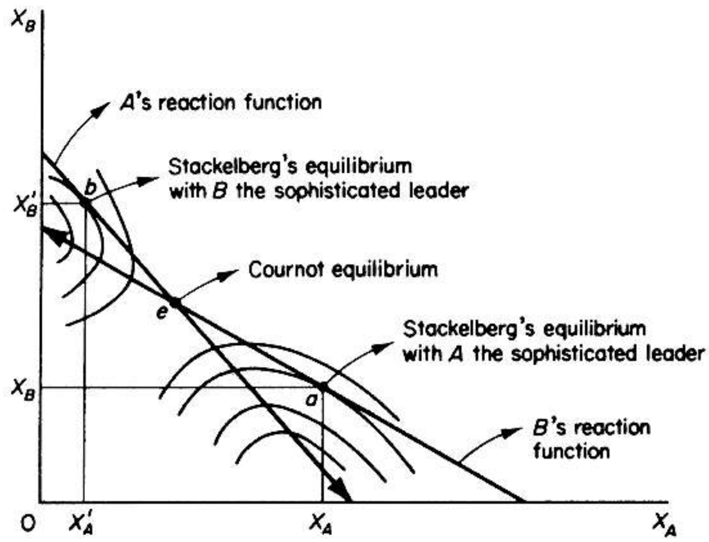


Figure 8

The sophisticated oligopolist becomes the leader and its rival becomes the follower. Compared to *Cournot* equilibrium, Firm-A is better off and its rival Firm-B is worse off because the former is producing nearer to its axis and the later is producing further from its own axis. If Firm-B is sophisticated oligopolist, it will choose X'_B corresponding to point *b* on Firm-A's *Reaction Curve*. Firm-B will be leader now and Firm-A becomes follower. Now Firm-B has the higher profit and its rival Firm-A has the lower profit in comparison to *Cournot* equilibrium. It means if only one firm is sophisticated, it will emerge as the leader and since it's rival is follower, there will be stable equilibrium.

If both are sophisticated, both will want to act as leaders to earn a greater profit. In this situation, market becomes unstable. There will either be a price war until one of the firm surrenders and agrees to act as a follower. Or a collusion between the firms may happen. In this case, both firms will abandon their *Reaction* functions and move to a point closer to or on the *Edgeworth's contract curve* and both will have higher profit.

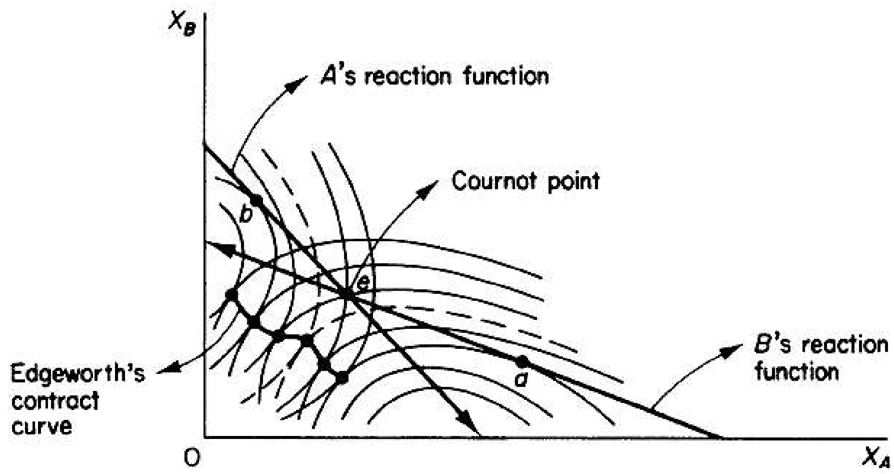


Figure 9

If the final' equilibrium lies on the *Edgeworth's contract curve* the joint profit of the industry are maximised (see Figure 9). This model shows that a collusive agreement becomes advantageous to both duopolists.

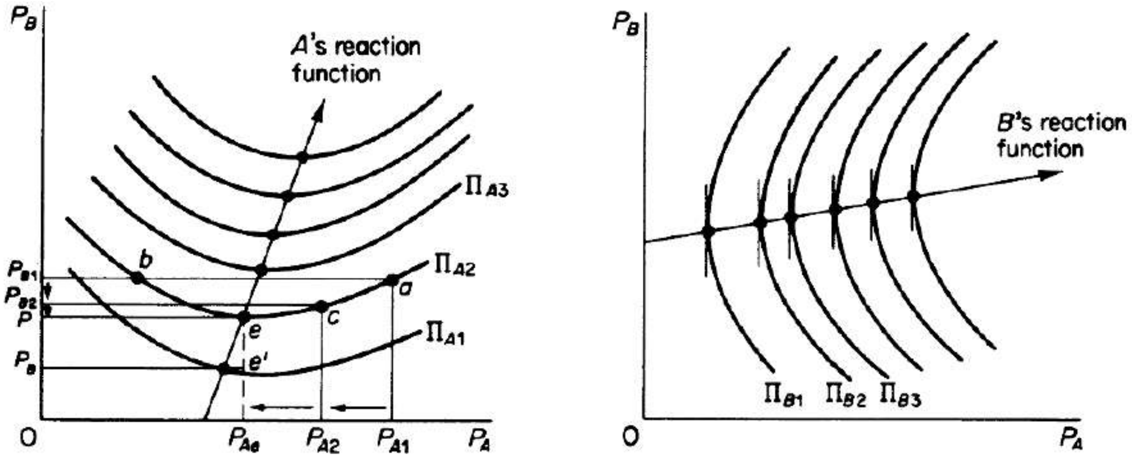
Stackelberg's model ends here.

2.4 *Bertrand* duopoly model

This model differs from that of *Cournot* in the assumption that each firm expects that his rival will keep its price constant, irrespective of its own decision about pricing. Thus, each firm faces the same market demand. In this model, the *Reaction Curves* are derived from *isoprofit* maps which are convex to the axes and on which prices of the duopolists are measured.

Each *isoprofit curve* (see Figure 10a and 10b) for a firm represents the same level of profit that each firm would accrue from various levels of prices charged by it and its rival.

The shape of *isoprofit curve* for Firm-A (see Figure 10a) shows that it must lower it's price up to the point *e* to match the reduction of product price of its competitors



(a) (b)

Figure 10

so as to maintain the level of its profit at Π_{A2}. However, if Firm-B continues to cut its prices, Firm-A will be unable to retain its profits, even if it continues to keep unchanged price at P_{Ae}. If Firm-B cuts its price at P_B, Firm-A will be in the lower *isoprofit curve* Π_{A2} which indicates lower profits. The reduction of profits of Firm-A is due to the fall in price, and the increase in output beyond the optimum level of utilisation of the plants which, in turn, increases costs. Thus, lower the *isoprofit*, lower is the level of profits. The *Reaction Curves* of Firm-B can be derived in a similar way.

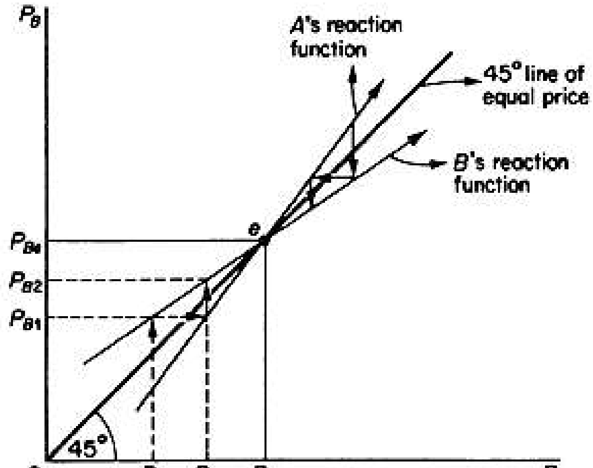


Figure 11

For any price charged by Firm-B there will be a unique price of Firm-A which maximises the latter's profit. This unique profit-maximising price is determined at the lowest point on the highest attainable *isoprofit curve* of Firm-A. The lowest point of the *isoprofit curves* lie to the right of each other because as Firm-A moves to a higher level of profit, it gains some of the customers of Firm-B when the latter increases its price, even if Firm-A also raises it's price. If we join the lowest points of the successive *isoprofit curves* we obtain the *Reaction Curve* of Firm-A.

Bertrand duopoly model leads to a stable equilibrium at e (see Figure 11). It is stable because any departure from it, sets in motion forces which will lead back to point e where the price charged by Firm-A and Firm-B are P_{Ae} and P_{Be}. It will be stable if Firm-A's *Reaction* function is steeper than that of Firm-B. If Firm-A charges a lower price P_{A1}, then Firm-B will charge P_{B2} . Firm-A will react to this decision of its rival by charging a higher price, P_{A2}. Firm-B will react by increasing its price and so on, until point e is reached, when the market will be in equilibrium. Similarly, if Firm-A charges price higher than P_{Ae}, competitive price cut takes place till equilibrium e is reached.

2.5 *Sweezy's* kinked demand curve model

P. Sweezy published an article in which he introduced the kinked demand curve as a tool for the determination of equilibrium in the oligopolist of markets. The demand curve of an oligopolist has a kink (point E in Figure 12) reflecting its behavioural pattern.

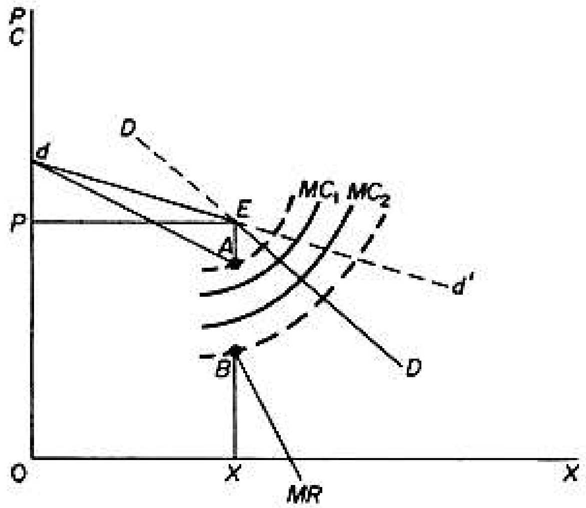


Figure 12

The upper section of the kinked demand curve has a high price elasticity than the lower part. Due to the kink in demand curve of oligopolist, its *MR* curve is discontinuous at the level of output corresponding to the kink. *MR* has two segments, *dA* corresponds to the upper part of the demand curve and the segment starting from *B* corresponds to the lower part of the kinked demand curve. The equilibrium for any firm is defined by the position of kink since at any point on the left of the kink *MC* is below *MR*, while to the right of the kink *MC* is larger than *MR*.

Thus, total profit is maximised at the point of kink. Here, equilibrium is not ensured by the intersection of *MC* and *MR* curves. In fact, *MC* passes through the disconnected segment *AB*. Although marginalistic calculation is behind 'kink-equilibrium', the kink demand curve is a manifestation of the breakdown of basic marginalistic rule (*MC*=*MR*).

The discontinuity of *MR* curve implies that there is a range within which a cost may change without affecting the equilibrium price and quantity. Thus, the kink can explain why price and output will not change despite changes in cost. Greater the difference of elasticities of the upper and lower parts of the kinked demand curve, wider the discontinuity in *MR* curve and wider the range of cost condition compatible with the equilibrium price and output. In an exceptional case, rise in cost is associated with the rise in price, for example, imposition of sales tax. It affects all firms equally. In that case, the point of kink shifts upward to the left, and equilibrium is established at a higher price and lower output (see Figure 13).

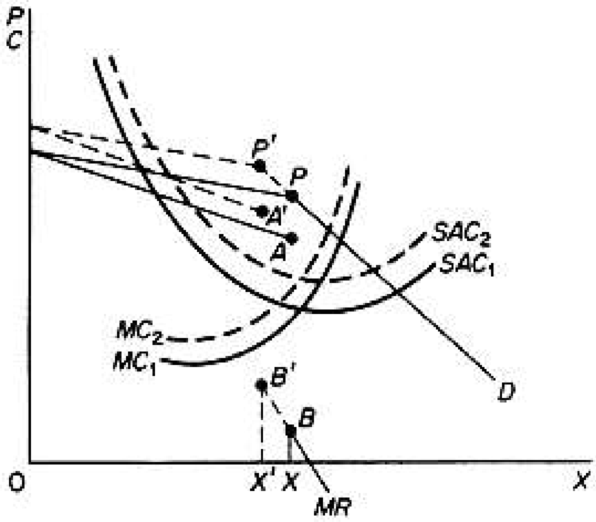


Figure 13

The independently running firms may move closer to a point of joint profit maximisation. In case of kinked demand curve, shift of market demands in upwards or downwards directions will affect the volume of output but not the level of price, so long as the cost curve passes through the range of discontinuity of the new *MR* curve (see Figure 14).

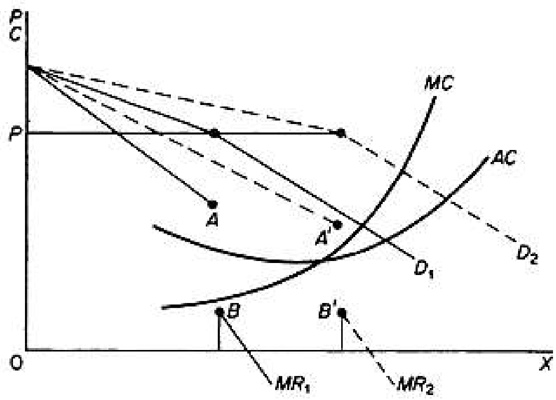


Figure 14

Here ends *Sweezy's* kinked demand curve model.

3 Game Theory

Von Neumann and Morgenstern demonstrated the relevance of game theory in description of economic behaviour. A game involves situation where activities of one player affect the welfare, profits, sales etc. of another player and vice versa. Such games are either cooperative or non-cooperative.

A cooperative game indicates that collusion between the two or more players will be mutually beneficial. Non-co-operative games indicate that a game can be played by one player to his own best interest. The benefits received from playing a game are called 'pay-offs'.

Actually games, co-operative or non-coperative, reflects oligopoly where individual firms may gain by adopting particular conflict strategies or all firms can gain mutually by adopting co-operative strategies. The game theories have cast light on some oligopoly problems. It's main achievement has been to restate the oligopoly theory in a somewhat attractive way.

Consider two person situation i.e. duopoly. We assume that the conflict between duopolists is about shares of the market. If Firm-A increases its share, Firm-B must reduce its share. A game in which Firm-A's gain are Firm-B's loss is called a zero sum game. As a result, zero-sum game contains no incentive for co-operation between two duopolists. Such game is called non-cooperative game.

Each firm is assumed to adopt strategies, viz. packaging, advertising, pricing policy and others to increase market share. We denote these strategies for Firm-A and Firm-B as A1, A2, A3 and B1, B2, B3. These strategies are shown in Figure 15.

B's Strategies

A's Strategies	B1	B2	B3
A1	50	30	10
A2	60	20	40
A3	90	80	30

Figure 15

Cells in the table are filled with numbers indicating the pay-off to Firm-A. Firm-Bs pay-off can be found by deducting Firm-As pay-off from the total available pay-off (in this case, size of the total market). Thus, if Firm-A selects strategy A3 and Firm-B selects strategy B1, Firm-As pay-off will be 9 units. Here, we are assuming it as 90% of market share. Then, Firm-B will secure 100% - 90% = 10% market share. This table is called pay-off matrix.

If we assume that Firm-A and Firm-B knows the relevant pay-off, we can consider which strategy each player will choose. If Firm-A chooses strategy A3, Firm-B will choose strategy B3 which will give it highest market share from cooperation, 100% - 30% = 70%. This is a non-zero-sum game. Similarly, A1 will be countered by B3 and A2 is countered by B2.

The gains from cooperation is a non-zero-sum gain shown in Figure 16.

B's Strategies

A's Strategies		B1	B2
	A1	3,5	0,6
	A2	6,0	1,2

Figure 16

Here, we assume that there are only two strategies for Firm-A and Firm-B. A1 and B1 means the first strategies of Firm-A and Firm-B. The combination of strategies of A1 and B1 yields a return of 3 to Firm-A and a return of 5 to Firm-B. It is not a zero-sum game because reward total in cells are not same. Now, Firm-A will select A2 since it will yield itself 6 if Firm-B selects B1 and 1 if Firm-B selects B2. This is better than the outcome if Firm-A selects A1.

Firm-B selects B2 because it promises maximum gain whatever strategy Firm-A selects. Hence, acting independently, the players choose A2 and B2 with combined gains of 3 units. But this is not a satisfactory solution from any point of view. A1, B1 gives maximum joint gains (8) compared to A2, B2. By moving from A2, B2 to A1, B1, Firm-A will gain 2 units and Firm-B will gain 3 units. To induce Firm-A to move to A1, Firm-B may have to offer a little more than the 2 units gain to Firm-A, even though Firm-A starts with a lower total. Firm-A may argue that it is unfair if the move secures 2 units for him but 3 units for Firm-B. It is still to Firm-Bs advantage to offer Firm-A bribe in addition to Firm-As automatic gains. If he gives him one-half of one unit, the result will be A = 3 + (1/2) and B = 4 + (1/2). The precise outcome will depend on bargaining strength and the type of rule adopted (equal gains, parity in the final total, improvement of relative position of one party to another and so on).

4 Collusive Oligopoly

The uncertainty in oligopolistic interdependence can be avoided if oligopolists come into collusive agreements. There are two main types of collusion: cartels and price leadership. These are secret agreements because open collusive action is illegal in most countries.

4.1 Cartels

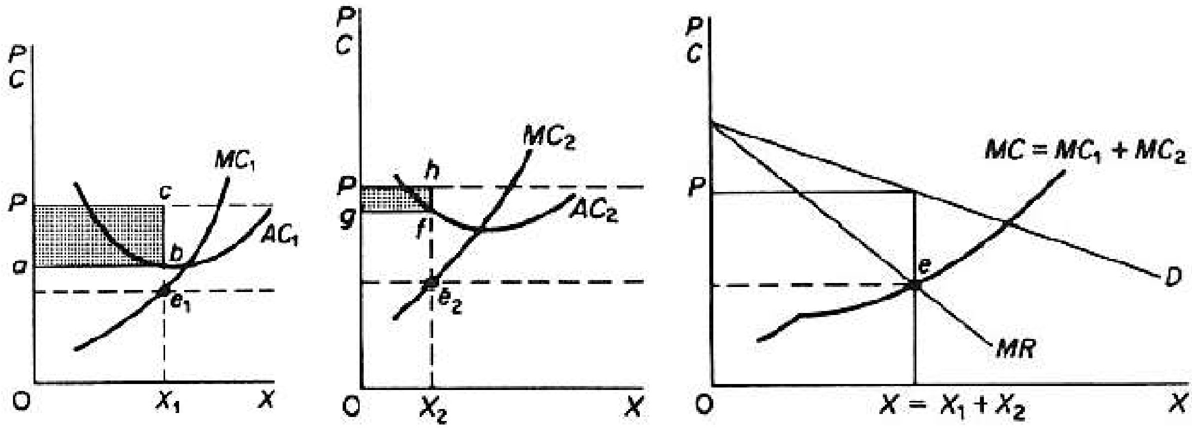
There are two types of cartels:

1. Cartel aiming at joint-profit maximisation

2. Cartel aiming at sharing the market

4.1.1 Cartel aiming at joint-profit maximisation

Cartel implies direct (secret) agreements among the competing oligopolists to reduce the uncertainty arising from their mutual interdependence. In first type, the aim of cartel is maximisation of the industry profit. This situation is identical to that of a multi-plant monopolist who seeks the maximisation of its profit. We assume that all firms are producing homogeneous product. The firms appoint a central agency to decide the total quantity and price which maximises group profits. It also decides the allocation of total production and maximum joint profit among the participating oligopolists. This central agency knows the costs of each firm, market demand curve and the corresponding *MR* curve.



(a) (b) (c)

Figure 17

We assume that there are only two firms in the cartel. The market *MC* curve is derived from the horizontal summation of the *MC* curves of individual firms shown in Figures 17a, 17b and **??**. The central agency, like multi-plant monopolist, will set the price at the intersection of industry *MR* and *MC* curves (point *e* in Figure 17c). The total output is*X* and it will be sold at price, *P*. Now the central agency allocates the production among Firm-A and Firm-B, by equating the MR to individual MCs. Thus, Firm-A will produce X_1 and Firm-B will produce X_2. The figures show that firm at lower cost produces a larger amount of output. The total profit is the sum of profits from output of the two firms.

The figures show joint profit maximisation theoretically. Practically there are many factors for which joint-profit maximisation may not be possible, such as:

1. In most cases, elasticity of market demand is underestimated. Each firm believes that the elasticity of its own demand curve is high due to the existence of perfect substitutes made by competitor, while the industry output is much less elastic.

2. The estimation of market MC may involve mistake due to incomplete knowledge of the individual MC curves.

3. Cartel agreements may take a long time to negotiate due to difference in size, costs, and markets of the individual firms.

4. Due to the stickiness of negotiated price, its level remains unchanged even if market conditions are changing.

5. Some firms may attempt to reduce price, to expand their sell before the final agreement.

6. The existence of high-cost firm which is higher than the equilibrium MC, will have to close if equilibrium is to be achieved (Figure 18).

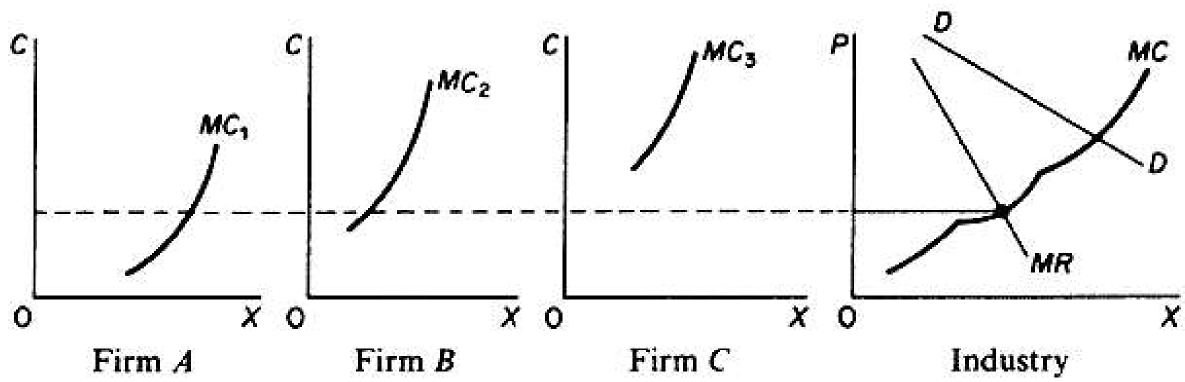


Figure 18

7. There is fear of government interference which may discourage cartel members to charge profit maximising price.

8. Due to fear of entry of other firms, cartel may not charge profit maximising price.

9. Even if firms adhere to the price defined by the central agency, they usually keep their freedom in deciding the style of their output and their selling activities which leads to increase cost and decrease monopoly profits.

4.1.2 Cartel aiming at sharing the market

There are two basic methods of sharing the market: non-price competition and determination of quotas.

Non-price competition: In this form of cartel, member firms agree on a common price, at which they can sell any quantity demanded. The price is set by bargaining. The low-cost firms bargain for a lower price where as high-cost firms for a higher price. The agreed price must be such as to allow some profits to all members. The firms are free to vary the quality, appearance of their product, advertising and other selling policies so as to attain a higher share of the market.

If all firms have the same costs, the price will be agreed at the monopoly level. With cost differences the cartel will be unstable, because the low-cost firms will have a strong incentive to break away from the cartel openly and charge a lower price or cheat the other members by secret price concessions to the buyers. Then others may split away from the cartel, and a price war and instability may develop until the fittest low-cost firms survive. Another possibility is that the members of cartel in conjunction may decide to start a price war until the cheated firm is driven out of business.

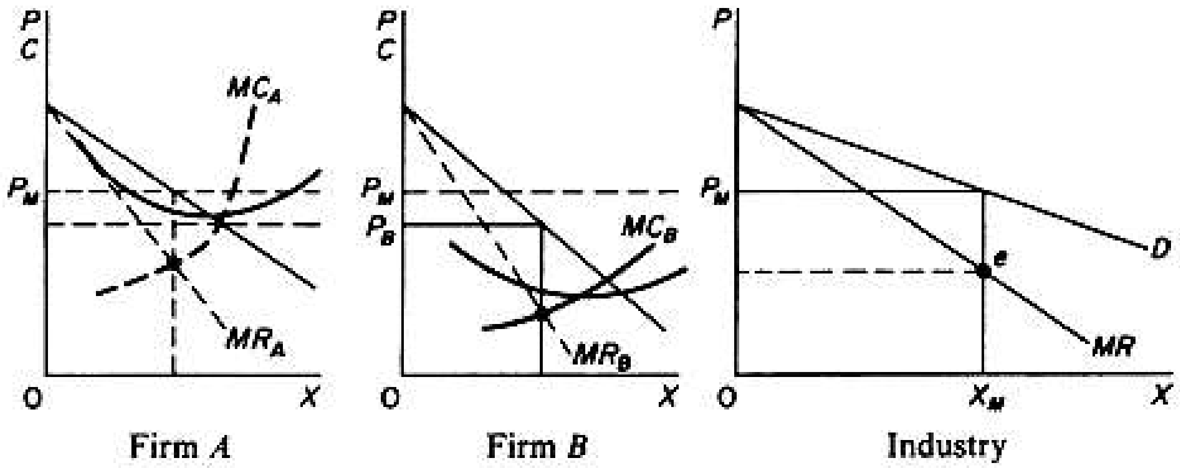


Figure 19

In Figure 19, Firm-B has lower costs than Firm-A. Hence, Firm-B will have the incentive to cut the price below monopoly level, thus driving the high-cost competitor, Firm-A, out of business.

Even with the same cost structure, these cartels are unstable because if one firm splits away and charges a slightly lower price than the monopoly price P_M while the others remain in the cartel, the splitting firm will attract a considerable number of customers from the others. Then all firms will have the same incentive to leave the cartel. This is how cartels become unstable unless supported by a tight legislation.

Sharing of the market by agreement on quotas: The second method for sharing a market is by agreement on quotas, i.e. an agreement on the quantity that each member may sell at an agreed price. If all firms have identical cost, the monopoly solution will emerge, with the market being shared equally by member firms.

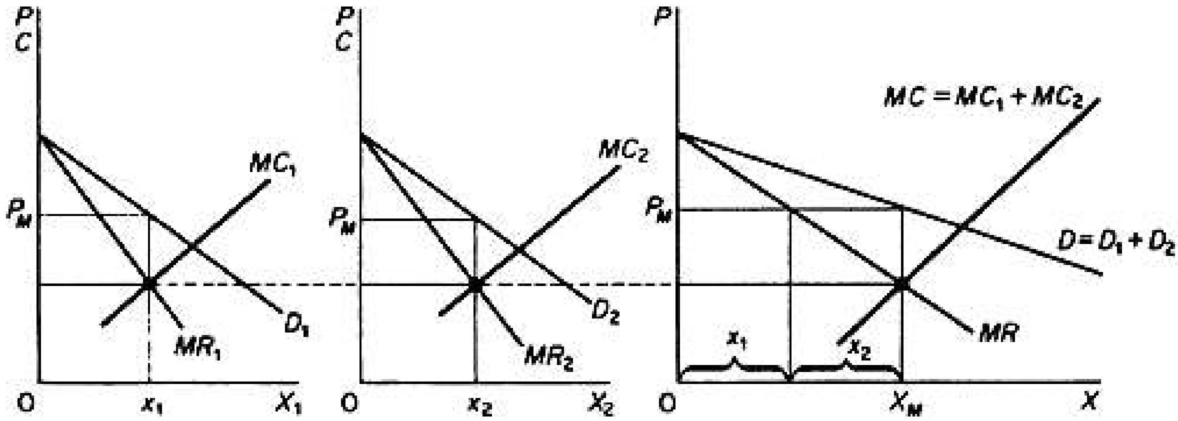


Figure 20

In Figure 20, the monopoly price is P_M and quota that will be agreed upon are $X_1 = X_2 = 1/2X_M$. If the costs are different, quotas and shares of the market will differ. Allocation of quota-shares on the basis of costs is also unstable. Shares in the case of cost differentials are decided by bargaining. During the bargaining process two main criteria are most often adopted, (a) past levels of sales and (b) productive capacity. Another popular method of sharing the market is by defining regions in which each firm will be allowed to sell. In such case of geographical sharing of a market, the price as well as the style of product may differ. This type of cartel may be unstable because agreements among the firms are often violated by the low-cost firms who always have the incentive to expand their output by selling at a lower price either openly or by secret price concessions or through advertising.

Cartel models of collusive oligopoly are closed models. If entry is free, the instability of cartel increases. If the profits of the cartel members are high and attract new firms in the industry, the newcomer will have a strong incentive not to join the cartel. This is because by not joining the cartel, his demand curve will be more elastic, and by charging a slightly lower price than the cartel, he can secure a considerable share in the market, provided cartel members stick to their agreement.

4.2 Price Leadership

Another form of collusion is price leadership. Here, one firm sets the price and the others follow it either because it is advantageous to them or because they prefer to avoid uncertainty about their competitors reactions. Price leadership may be practised either by implicit agreement or informally since open collusive agreements are illegal in most countries.

Price leadership is more widespread than cartels, because it allows the members complete freedom regarding their product and selling activities. If the product is homogeneous and the firms are highly concentrated in a location, the price will be identical. However, if the product is differentiated, prices will differ, but the direction of their change will be the same.

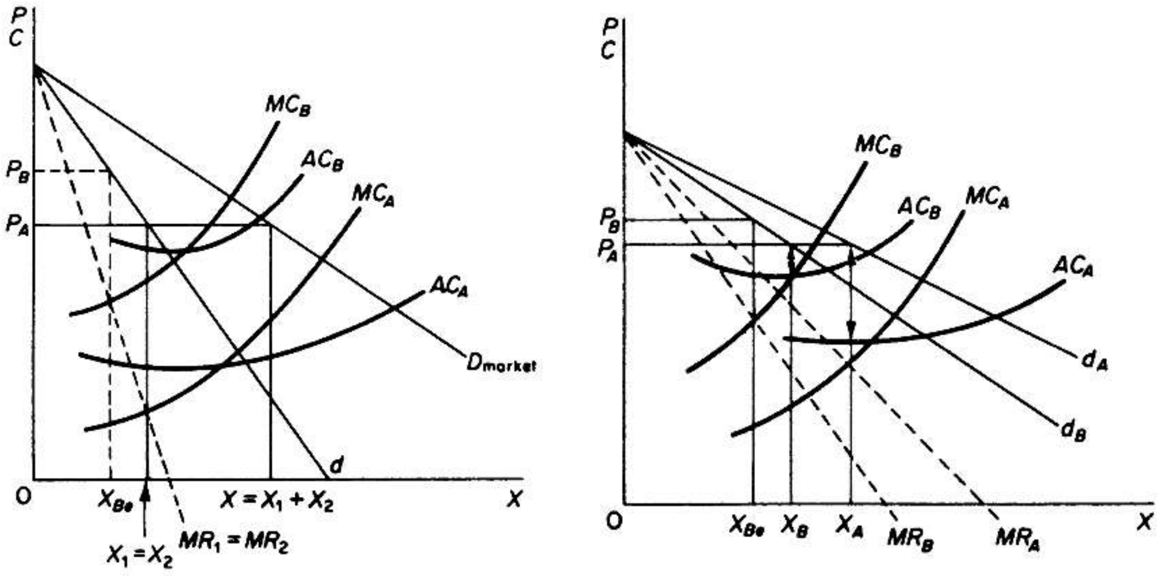
There are various forms of price leadership. The most common types of leadership are:

1. Price leadership by a low-cost firm

2. Price leadership by a large dominant firm

4.2.1 Model of low-cost firm as a price leader

Consider a duopoly market, with two firms which produce a homogeneous product at different costs. The firms may have equal markets or they may come to an agreement to share the market equally (Figure 21a) or they may have unequal markets (Figure 21b).



(a) Low-cost price leader firms with equal market shares

(b) Low-cost price leader firms with unequal market shares

Figure 21

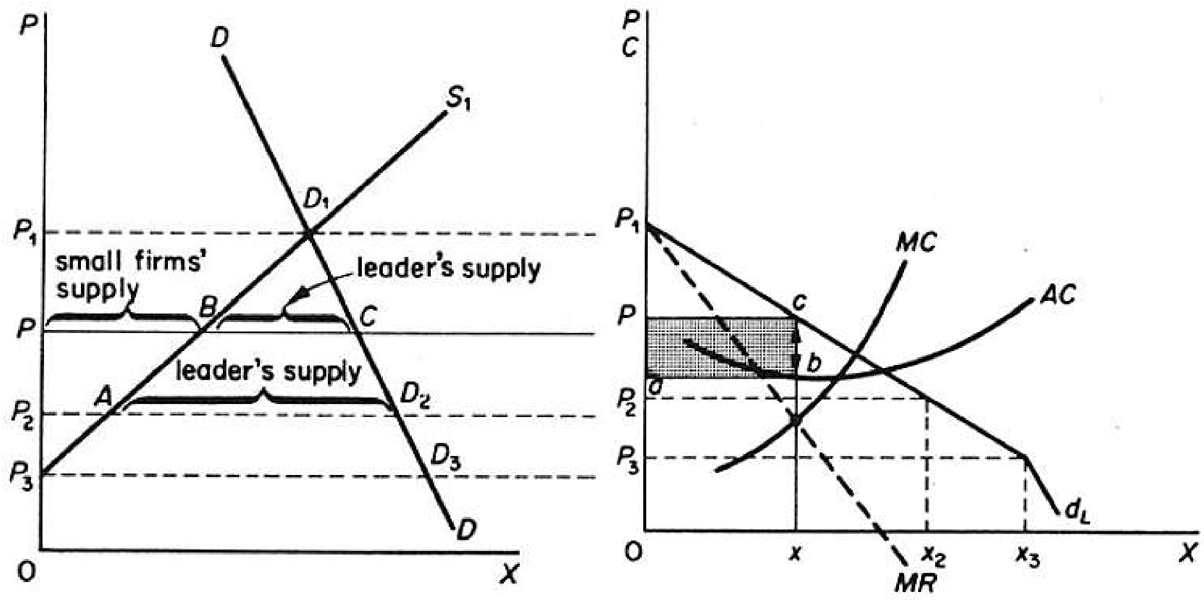
The firm with lower cost will charge a lower price P_A and this price will be followed by the high-cost firm, although at this price Firm-B (the follower) does not maximise its profit. The follower will obtain a higher profit by producing a lower output X_Be and selling it at a higher price P_B. However, it prefers to follow the leader, sacrificing some of its profit to avoid a price war. This implies that the follower must supply a quantity

(OX_B in Figure 21b or OX_1=OX_2 in Figure 21a) that is sufficient to maintain the price set by the leader.

Although the price-leadership model states that the leader sets a price and the follower adopts it, the firms must enter a share of the market agreement, formally or informally. If this agreement is not made, the follower could adopt the price of the leader but produce a lower quantity than the level required to maintain the leader determined price in the market. In this way, the follower can push the leader into non-profit maximising position.

4.2.2 Model of dominant firm as a price leader

In this model, it is assumed that there is a large dominant firm which has a considerable share of the market and some smaller firms having small market share each. The market demand (*DD* in Figure 22a) is assumed to be known to the dominant firm.



(a) (b)

Figure 22

It is also assumed that the dominant leader knows about the *MC* curves of the smaller firms, which he can add horizontally to get the total supply by the small firms at each price or he has a fair estimation of this supply from its past experience.

At each price, the large firm is able to supply the part of total market demand which is not supplied by the smaller firms. In Figure 22a, at each price, the demand for the product of the leader is the difference between total D and the total supply S_1. At price P_1, the demand for the product of the leader is zero. At P_2, the part P_2A is supplied by the small firms and the remaining AD_2 is supplied by the leader. Below P_3, the market demand coincides with the leaders demand curve.

Having derived its demand curve (d_L in Figure 22b) and given its MC curve the dominant firm will set the price P at which its $MR{=}MC$ and its output is Ox. At price P, the total market demand is PC and the part PB is supplied by the small firms followers while quantity $BC{=}Ox$ is supplied by the leader.

As small firms are price takers they may or may not maximise their profit. It is assumed that the small firms can not sell more than the quantity denoted by S_1. However, if the leader is to maximise its profit, it must ensure that the small firms will not only follow its price, but that they will also produce the right quantity (PB, at price P). Thus, if there is no tight sharing-the-market agreement, the small firms may produce less output than PB and thus force the leader to a non-maximising position.

The price-leadership model will lead to a stable equilibrium if the leader has the power to make the other firms in the industry follow his price increase or price decrease, provided that there is some agreement for sharing the market. In order to have the power to impose its price, the leader must be both a low-cost and a large firm. If a firm has low costs but is very small compared to the leader, it may not find it possible to survive a price, or advertising or product design war that the dominant firm may start. On the other hand, if the dominant firm looses its cost advantage, it will also lose its power to impose a price increase, since the smaller firms having lower costs, will normally not follow it in price increases.

5 Appendices

5.A Mathematical derivation of the reaction curves: A mathematical version of *Cournot* model

Assume that the market demand facing the duopolists is

$$X = a^* + b^*P$$

or

$$P = a + bX \qquad b < 0$$

Given that $X = X_1 + X_2$

$$\frac{\partial X}{\partial X_1} = \frac{\partial X}{\partial X_2} = 1$$

and the MRs of the duopolists need not be the same. Actually if the duopolists are of unequal size the one with the larger output will have the smaller MR.

Proof:

$$R_i = pX_i$$

$$p = a + b(X_1 + X_2) = f(X_1, X_2)$$

Thus

$$\frac{\partial R_i}{\partial X_i} = p + X_i \frac{\partial P}{\partial X_i}$$

But

$$\frac{\partial P}{\partial X_1} = \frac{\partial P}{\partial X_2} = \frac{\partial P}{\partial X} = b$$

Therefore

$$\frac{\partial R_i}{\partial X_i} = P + X_i \frac{\partial P}{\partial X} = P + (X_i)(b)$$

Given that $P > 0$ while $b < 0$, it is clear that the larger X_i is, the smaller the MR will be.

The two duopolists have different costs

$$C_1 = f_1(X_1) \quad \text{and} \quad C_2 = f_2(X_2)$$

The first duopolist maximises his profit by assuming X_2 constant, irrespective of his own decisions, while the second duopolist maximises his profit by assuming that X_1 will remain constant.

The first-order condition for maximum profits of each duopolist is

$$\left.\begin{aligned} \frac{\partial \Pi_1}{\partial X_1} = \frac{\partial R_1}{\partial X_1} - \frac{\partial C_1}{\partial X_1} = 0 \\ \frac{\partial \Pi_2}{\partial X_2} = \frac{\partial R_2}{\partial X_2} - \frac{\partial C_2}{\partial X_2} = 0 \end{aligned}\right\} \tag{9.1}$$

Rearranging we have

$$\left.\begin{aligned} \frac{\partial R_1}{\partial X_1} = \frac{\partial C_1}{\partial X_1} \\ \frac{\partial R_2}{\partial X_2} = \frac{\partial C_2}{\partial X_2} \end{aligned}\right\} \tag{9.2}$$

Solving the first equation of (9.2) for X_1 we obtain X_1 as a function of X_2, that is, we obtain the reaction curve of firm A. It expresses the output which A must produce in order to maximise his profit for any given amount X_2 of his rival.

Solving the second equation of (9.2) for X_2 we obtain X_2 as a function of X_1, that is, we obtain the reaction function of firm B.

If we solve the two equations simultaneously we obtain the Cournot equilibrium, the values of X_1 and X_2 which satisfy both equations; this is the point of intersection of the two reaction curves.

The second-order condition for equilibrium requires that

$$\frac{\partial^2 \Pi_i}{\partial X_i^2} = \frac{\partial^2 R_i}{\partial X_i^2} - \frac{\partial^2 C_i}{\partial X_i^2} < 0 \qquad (i = 1, 2)$$

or

$$\frac{\partial^2 R_i}{\partial X_i^2} < \frac{\partial^2 C_i}{\partial X_i^2}$$

Each duopolist's MR must be increasing less rapidly than his MC, that is, the MC must cut the MR from below, for both duopolists.

A numerical example

Assume that the market demand and the costs of the duopolists are

$$P = 100 - 0{\cdot}5(X_1 + X_2)$$
$$C_1 = 5X_1$$
$$C_2 = 0{\cdot}5X_2^2$$

The profits of the duopolists are

(a) $$\Pi_1 = PX_1 - C_1 = [100 - 0{\cdot}5(X_1 + X_2)]X_1 - 5X_1$$

or

$$\Pi_1 = 100X_1 - 0{\cdot}5X_1^2 - 0{\cdot}5X_1X_2 - 5X_1$$

(b) $$\Pi_2 = PX_2 - C_2 = [100 - 0{\cdot}5(X_1 + X_2)]X_2 - 0{\cdot}5X_2^2$$

or

$$\Pi_2 = 100X_2 - 0{\cdot}5X_2^2 - 0{\cdot}5X_1X_2 - 0{\cdot}5X_2^2$$

Collecting terms we have

$$\Pi_1 = 95X_1 - 0{\cdot}5X_1^2 - 0{\cdot}5X_1X_2$$

and

$$\Pi_2 = 100X_2 - X_2^2 - 0{\cdot}5X_1X_2$$

For profit maximisation under the Cournot assumption we have

$$\left.\begin{aligned} \frac{\partial \Pi_1}{\partial X_1} &= 0 = 95 - X_1 - 0{\cdot}5X_2 \\ \frac{\partial \Pi_2}{\partial X_2} &= 0 = 100 - 2X_2 - 0{\cdot}5X_1 \end{aligned}\right\}$$

The reaction functions are

$$X_1 = 95 - 0{\cdot}5X_2$$
$$X_2 = 50 - 0{\cdot}25X_1$$

The graphical solution of Cournot's model is found by the intersection of the two reaction curves which are plotted in figure 9.10.

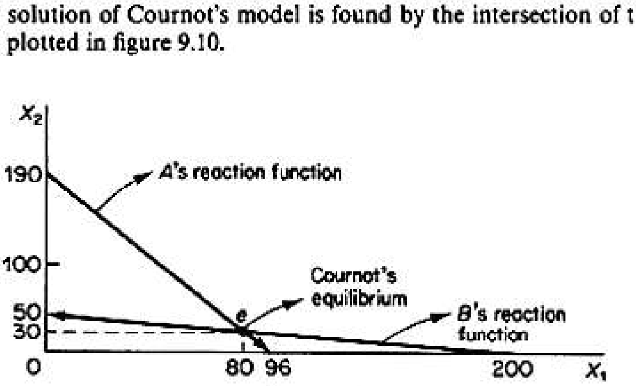


Figure 9.10

Mathematically the solution of system (9.3) yields

$$\left.\begin{aligned} X_1 &= 95 - 0{\cdot}5X_2 \\ X_2 &= 50 - 0{\cdot}25X_1 \end{aligned}\right\}$$

$$X_1 = 95 - 0{\cdot}5(50 - 0{\cdot}25X_1)$$

or

$$X_1 = 80$$

and

$$X_2 = 50 - 0{\cdot}25X_1 = 50 - (0{\cdot}25)(80) = 30$$

Thus total output in the market is

$$X = X_1 + X_2 = 120$$

and the market price

$$P = 100 - 0{\cdot}5X = 45$$

Note that

$$MR_1 = \frac{\partial R_1}{\partial X_1} = \frac{\partial (PX_1)}{\partial X_1} = P + X_1\frac{\partial P}{\partial X}$$

$$MR_1 = 45 + 80(-0{\cdot}5)$$

$$MR_1 = 5$$

while $MR_2 = 45 + 30(-0{\cdot}5)$

$$MR_2 = 30$$

That is, the firm with the larger output has the smaller marginal revenue. The profits of the duopolists are

$$\Pi_1 = PX_1 - C_1$$

$$\Pi_1 = (45)(80) - 5(80) = 3200$$

and

$$\Pi_2 = PX_2 - C_2$$

$$\Pi_2 = (45)(30) - 0{\cdot}25(30)^2 = 900$$

The second-order condition is satisfied for both duopolists:

$$\left.\begin{aligned} \frac{\partial \Pi_1}{\partial X_1} &= 95 - X_1 - 0{\cdot}5X_2 & \qquad \frac{\partial \Pi_2}{\partial X_2} &= 100 - 2X_2 - 0{\cdot}5X_1 \\ \frac{\partial^2 \Pi_1}{\partial X_1^2} &= -1 < 0 & \qquad \frac{\partial^2 \Pi_2}{\partial X_2^2} &= -2 < 0 \end{aligned}\right\}$$

5.B A numerical example of *Stackelberg's* model

Assume that in a duopoly market the demand function is

$$P = 100 - 0{\cdot}5(X_1 + X_2)$$

and the duopolists' costs are

$$C_1 = 5X_1 \qquad \text{and} \qquad C_2 = 0{\cdot}5X_2^2$$

The reaction functions are found by taking the partial derivatives of the duopolists' profit functions and equating them to zero:

$$\Pi_1 = PX_1 - C_1 = 95X_1 - 0{\cdot}5X_1^2 - 0{\cdot}5X_1X_2$$
$$\Pi_2 = PX_2 - C_2 = 100X_2 - X_2^2 - 0{\cdot}5X_1X_2$$

The partial derivatives are

$$\frac{\partial \Pi_1}{\partial X_1} = 95 - X_1 - 0{\cdot}5X_2 = 0$$

$$\frac{\partial \Pi_2}{\partial X_2} = 100 - 2X_2 - 0{\cdot}5X_1 = 0$$

The reaction functions are

$$X_1 = 95 - 0{\cdot}5X_2 \rightarrow A\text{'s reaction curve}$$
$$X_2 = 50 - 0{\cdot}25X_1 \rightarrow B\text{'s reaction curve}$$

(1) *Stackelberg's solution with* A *being the sophisticated leader*

Firm *A* will substitute *B*'s reaction function in its own profit equation, which it will then maximise as if it were a monopolist:

$$\Pi_1 = PX_1 - C_1 = 95X_1 - 0{\cdot}5X_1^2 - 0{\cdot}5X_1X_2$$

Substitute $X_2 = 50 - 0{\cdot}25X_1$

Maximise $\Pi_1 = 70X_1 - 0{\cdot}375X_1^2$

(*a*) First-order condition: $\dfrac{\partial \Pi_1}{\partial X_1} = 70 - 0{\cdot}75X_1 = 0$

This yields output: $X_1 = 93\frac{1}{3}$

and profit: $\Pi_1 = 70X_1 - 0{\cdot}375X_1 = 3267$

(*b*) The second-order condition for profit maximisation is fulfilled.

Firm *B* would be the follower. It would assume that *A* would produce $93\frac{1}{3}$ units; thus *B* substitutes this amount in its reaction function

$$X_2 = 50 - 0{\cdot}25X_1 = 26\tfrac{2}{3}$$

and its profit would be

$$\Pi_2 = 100X_2 - X_2^2 - 0{\cdot}5X_1X_2 = 155{\cdot}5$$

(2) *Stackelberg's solution if firm* B *is the sophisticated duopolist*

Firm *B* will substitute *A*'s reaction function in its own profit function, and it will proceed to maximise this profit as a monopolist

$$\Pi_2 = PX_2 - C_2 = 100X_2 - X_2^2 - 0{\cdot}5X_1X_2$$

Substitute $X_1 = 95 - 0{\cdot}5X_2$ (i.e., *A*'s reaction function)

$$\Pi_2 = 52{\cdot}5X_2 - 0{\cdot}75X_2^2$$

(*a*) The first-order condition for the maximisation of Π_2 requires

$$\frac{\partial \Pi_2}{\partial X_2} = 52{\cdot}5 - 1{\cdot}5X_2 = 0$$

which yields output: $X_2 = 35$

and profit: $\Pi_2 = 52{\cdot}5X_2 - 0{\cdot}75X_2^2 = 918{\cdot}75$

(*b*) The second-order condition for the maximisation of Π_2 is fulfilled.

The follower is now firm *A* which will act on the Cournot assumption; it will assume that the rival will keep his quantity at $X_2 = 35$, and will find its own output by substituting this quantity in its reaction function.

$$X_1 = 95 - 0{\cdot}5X_2 = 77{\cdot}5$$

and its profit is

$$\Pi_1 = 95X_1 - 0{\cdot}5X_1^2 - 0{\cdot}5X_1X_2 = 3003$$

(3) *Stackelberg's disequilibrium*

If both entrepreneurs adopt Stackelberg's sophisticated pattern of behaviour, each will examine his profits if he acts as a leader and if he acts as a follower, and will adopt the action that will yield him the greatest profit.

Firm A calculates its profits both as a leader and as a follower:

If *A* is the leader his profits are 3267

If *A* is the follower his profits are 3003

Clearly firm *A* will prefer to act as the leader.

Firm B similarly, calculates its profits as a leader and as a follower:

If *B* is the leader his profits are 918·75

If *B* acts as the follower his profits are 155·50

Thus firm *B* will also choose to act as the leader.[1]

With both firms acting in the sophisticated way implied by Stackelberg's behavioural hypothesis both will want to act as leaders. As they attempt to do so they find that their expectations about the rival are not fulfilled and 'warfare' will start, unless they decide to come to a collusive agreement.

5.C A mathematical presentation of the joint profit maximisation cartel model

The mathematical presentation of the cartel model which aims at joint profit maximisation is identical to that of the multiplant monopolist. Thus:

$$\text{Maximise} \quad \Pi = \Pi_1 + \Pi_2$$

given

$$P = f(X) = f(X_1 + X_2)$$
$$C_1 = f_1(X_1)$$
$$C_2 = f_2(X_2)$$

We have

$$\Pi_1 = R_1 - C_1$$
$$\Pi_2 = R_2 - C_2$$

Therefore

$$\Pi = R_1 + R_2 - C_1 - C_2 = R - C_1 - C_2$$

and the market marginal revenue is

$$\frac{\partial R}{\partial X} = \frac{\partial R}{\partial X_1} = \frac{\partial R}{\partial X_2}$$

that is, each additional unit will bring the same MR, irrespective of the plant in which it is produced, since all units of X are sold at the same price P.

The first-order condition for maximisation of the joint profit Π requires the allocation of output in such a way that the MC of each firm is the same:

$$\frac{\partial \Pi}{\partial X_1} = \frac{\partial R}{\partial X} - \frac{\partial C_1}{\partial X_1} = 0 \rightarrow \frac{\partial R}{\partial X} = \frac{\partial C_1}{\partial X_1}$$

$$\frac{\partial \Pi}{\partial X_2} = \frac{\partial R}{\partial X} - \frac{\partial C_2}{\partial X_2} = 0 \rightarrow \frac{\partial R}{\partial X} = \frac{\partial C_2}{\partial X_2}$$

or

$$MR = MC_1 = MC_2$$

The second-order condition for maximisation of joint Π requires

$$\frac{\partial^2 R}{\partial X^2} < \frac{\partial^2 C_1}{\partial X_1^2} \quad \text{and} \quad \frac{\partial^2 R}{\partial X^2} < \frac{\partial^2 C_2}{\partial X_2^2}$$

that is, the MC of each firm must be increasing faster than the (common) MR of the output o the cartel as a whole.

These are the same equilibrium conditions as the ones we derived in Chapter 6 for the multiplan monopolist.

Numerical Example

Assume that the market demand is

$$P = 100 - 0{\cdot}5(X) = 100 - 0{\cdot}5(X_1 + X_2) \tag{10.1}$$

and that the two colluding firms have costs given by

$$C_1 = 5X_1, \quad \text{and} \quad C_2 = 0{\cdot}5X_2^2$$

The central agency of the cartel aims at the maximisation of total profit

$$\Pi = \Pi_1 + \Pi_2$$

where

$$\Pi_1 = R_1 - C_1 \quad \text{and} \quad \Pi_2 = R_2 - C_2$$

Thus

$$\Pi = (R_1 + R_2) - C_1 - C_2$$
$$\Pi = P(X_1 + X_2) - C_1 - C_2$$
$$\Pi = [100 - 0{\cdot}5(X_1 + X_2)](X_1 + X_2) - 5X_1 - 0{\cdot}5X_2$$
$$\Pi = 95X_1 - 100X_2 - 0{\cdot}5X_1^2 - X_2^2 - X_1X_2 \tag{10.2}$$

Setting the partial derivatives equal to zero we obtain

$$\frac{\partial\Pi}{\partial X_1} = 85 - X_1 - X_2 = 0$$

$$\frac{\partial\Pi}{\partial X_2} = 100 - X_1 - 2X_2 = 0$$

Solving for X_1 and X_2 we find

$$X_1 = 90 \quad \text{and} \quad X_2 = 5$$

The price is found by substituting in (10.1)

$$P = 100 - 0{\cdot}5(X_1 + X_2) = 52{\cdot}5$$

The joint profit may be obtained by substituting in (10.2)

$$\Pi = 95(90) - 100(5) - 0{\cdot}5(90)^2 - (5)^2 - (90)(5) = 4525$$

5.D A numerical example of the price-leadership models

A numerical example of the price-leadership models

1. *The low-cost price leader*

It is assumed, for simplicity, that there are only two firms in the industry.
The market demand is defined by the function

$$P = a - b(X) = a - b(X_1 + X_2)$$

where X_1 = output of firm A
X_2 = output of firm B

The firms have different costs, defined by the functions

$$C_1 = f_1(X_1)$$
$$C_2 = f_2(X_2)$$

where $C_1 < C_2$.

The leader will be the low-cost firm A. He assumes that the rival firm will produce an equal amount of output to his own, that is

$$X_1 = X_2$$

With this assumption, the demand function relevant to the leader's decision is

$$P = a - 2b(X_1)$$

The low-cost leader will set the price which maximizes his own profit

$$\Pi_1 = R_1 - C_1 = PX_1 - C_1$$

or

$$\Pi_1 = (a - 2bX_1)X_1 - C_1$$

The first-order condition for the maximisation of Π_1 requires

$$\frac{\partial \Pi_1}{\partial X_1} = \frac{\partial R_1}{\partial X_1} - \frac{\partial C_1}{\partial X_1} = 0$$

or

$$\frac{\partial R_1}{\partial X_1} = \frac{\partial C_1}{\partial X_1}$$

that is, $MR = MC$.
The second-order condition requires

$$\frac{\partial^2 \Pi_1}{\partial X_1^2} < 0$$

or

$$\frac{\partial^2 R_1}{\partial X_1^2} < \frac{\partial^2 C_1}{\partial X_1^2}$$

that is, the marginal cost must rise faster than the marginal revenue; or the MC must cut the MR curve from below.

The solution of this problem yields the price P and output X_1 that the leader must produce in order to maximise his profit. The follower will adopt the same price and will produce (*ex hypothesi*) an equal amount of output ($X_2 = X_1$). Given that $C_2 > C_1$, the follower does not maximise his profit. He would prefer (under the above assumptions) to produce a lower level of output and sell it at a higher price.

A numerical example:

Assume that the market demand is

$$P = 105 - 2{\cdot}5X = 105 - 2.5(X_1 + X_2)$$

The cost functions of the two firms are

$$C_1 = 5X_1$$
$$C_2 = 15X_2$$

The leader will be the low-cost firm *A*: he will set a price which will maximise his own profit on the assumption that the rival firm will adopt the same price and will produce an equal amount of output. Thus the demand function relevant to the leader's decision is

$$P = 105 - 2{\cdot}5(2X_1) = 105 - 5X_1$$

and his profit function is

$$\Pi_1 = R_1 - C_1 = PX_1 = (105 - 5X_1)X_1 - 5X_1$$

or

$$\Pi_1 = 100X_1 - 5X_1^2$$

From the first-order condition we have

$$\frac{\partial \Pi_1}{\partial X_1} = 100 - 10X_1 = 0$$

which yields

$$X_1 = 10$$

Substituting in the price equation, we find

$$P = 105 - 5X_1 = 55$$

The follower will adopt the same price (55) and will produce an equal level of output ($X_2 =$ 10). Note that the profit-maximising output of firm *B* would be $X_2^* = 9$ units, and he would sell it at $P^* = 60$. This solution is found by maximising firm *B*'s profit function

$$\Pi_2 = R_2 - C_2 = (105 - 5X_2)X_2 - 15X_2$$

2. *The dominant-firm leader*

This model is also called 'the partial monopoly' model since the large firm acts as a monopolist while the small firms are price-takers and act like the firms in pure competition.

This model is a combination of the theory of pure competition and the theory of monopoly.

(*a*) The small firms are assumed to accept the prevailing price and adjust their output levels to maximise profit like a perfectly competitive firm.

(*b*) The dominant firm knows the *MC* curves of the small firms. By adding these curves horizontally it obtains the supply curve S_1 of the small firms as a function of price. Assume

$$S = 0{\cdot}2P$$

(*c*) The dominant firm is assumed to know the market demand

$$D = 50 - 0{\cdot}3P$$

Thus the dominant firm derives its own demand curve as the difference $X = D - S$ at any one price

$$X = D - S$$
$$X = 50 - 0{\cdot}3P - 0{\cdot}2P$$

or

$$P = 100 - 2X$$

(*d*) The dominant firm maximises its own profit given its total cost function

$$C = 2X$$

Thus

$$\Pi = R - C = PX - 2X$$
$$\Pi = (100 - 2X)X - 2X = 98X - 2X^2$$
$$\frac{\partial \Pi}{\partial X} = 98 - 4X = 0$$

and

$$X = 24{\cdot}5$$

The leader will set the price

$$P = 100 - 2X = 100 - 2(24{\cdot}5) = 51$$

At this price the market is in equilibrium. The total quantity demanded is

$$D = 50 - 0{\cdot}3P = 34{\cdot}7$$

The total demand of 34·7 units of output (at the price 51 set by the leader) is covered by the leader who produces $X = 24{\cdot}5$ units and the small firms who produce the remainder

$$S_1 = 0{\cdot}2P = 0{\cdot}2(51) = 10{\cdot}2$$

3. *The market-sharing firm leader*

In this model the product is homogeneous. The firms agree that they are going to share the market in constant proportions:

$$k_1 = \frac{X_1}{X} \qquad \text{and} \qquad k_2 = \frac{X_2}{X}$$

Given that $X = X_1 + X_2$ we may rewrite the shares

$$k_1 = \frac{X_1}{X_1 + X_2} \qquad \text{and} \qquad k_2 = \frac{X_2}{X_1 + X_2}$$

Clearly $k_1 + k_2 = 1$, and hence $k_2 = (1 - k_1)$, or $k_1 = (1 - k_2)$. Thus the reaction functions of the two firms are

Firm A	*Firm B*
$k_1(X_1 + X_2) = X_1$	$k_2(X_1 + X_2) = X_2$
$X_1(1 - k_1) = k_1 X_2$	$X_2(1 - k_2) = k_2 X_1$
$X_1 = \frac{k_1 X_2}{(1 - k_1)}$	$X_2 = \frac{k_2 X_1}{(1 - k_2)}$

Assume that the firms agree that firm *A* will be the leader. Firm *A* knows its own demand curve and its own costs:

$$P_1 = 100 - 2X_1 - X_2$$
$$C_1 = 2{\cdot}5X_1^2$$

The leader will set his price so as to maximise his profit, on the assumption that B will be the follower and will react according to his reaction function $X_2 = k_2 X_1/(1 - k_2)$, adjusting his output whenever the leader changes his, so that the shares remain at the agreed constant levels. Assume that $k_1 = \frac{2}{3}$ and $k_2 = \frac{1}{3}$. Thus $X_2 = X_1/[3(1 - \frac{1}{3})] = 0{\cdot}5X_1$.

The profit function of the leader is

$$\Pi_1 = R_1 - C_1 = P_1 X_1 - C_1$$
$$\Pi_1 = (100 - 2X_1 - X_2)X_1 - 2{\cdot}5X_1^2$$

Substituting the reaction function of firm B

$$X_2 = 0{\cdot}5X_1$$

we obtain the profit function of the leader

$$\Pi_1 = (100 - 2X_1 - 0{\cdot}5X_1)X_1 - 2{\cdot}5X^2 = 100X_1 - 5X_1^2$$

For the maximisation of the leader's profit we have

$$\frac{\partial \Pi_1}{\partial X_1} = 100 - 10X_1 = 0$$
$$X_1 = 10$$

The leader will set the profit-maximising price

$$\begin{aligned} P_1 &= 100 - 2X_1 - X_2 \\ &= 100 - 2X_1 - 0{\cdot}5X_1 \\ &= 100 - 2{\cdot}5X_1 = 75 \end{aligned}$$

The leader's profit is

$$\Pi_1 = 100X_1 - 5X_1^2 = 1000 - 500 = 500$$

The quantity which will be produced by the follower is

$$X_2 = 0{\cdot}5X_1 = 5$$

and he will sell it at the price of the leader $P_1 = 75$. Thus in the sharing-market model of leadership the firms agree on the shares and on who will be the leader. The leader maximises his own profit, by substituting in his profit function the share-reaction curve of the follower. This implies that the leader sets the quantity that maximises his own profit, on the assumption that the follower will adjust his own quantity on the basis of the agreed shares.