VIVEKANANDA COLLEGE
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NAAC ACCREDITED 'A' GRADE

Topic: Grand Canonical Ensemble I (https://youtu.be/1uogRCJXwmM)

Course Title: Statistical Mechanics

Paper:PHY 423

Unit: 2

Semester: 2

Name of the Teacher:Arvind Pan

Name of the Department:Physics

Grand Canonical Ensemble

Microcanonical system

N
E
V

$\Delta N = 0$
$\Delta E = 0$
$\Delta V = 0$

Canonical Ensemble

System | Reservoir

N
V
E ⟷ $\Delta E \neq 0$
T

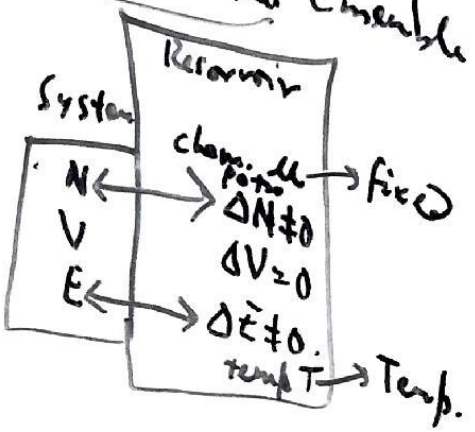
fixed

diathermic
$\Delta N = 0$
$\Delta V = 0$
$\Delta E \neq 0$

Impermeable Membrane between System & reservoir.
fixed walls

Partition function
$Q(N, V, T)$

Grand Canonical Ensemble



$\mathcal{Q} = \mathcal{Q}(\mu, V, T)$

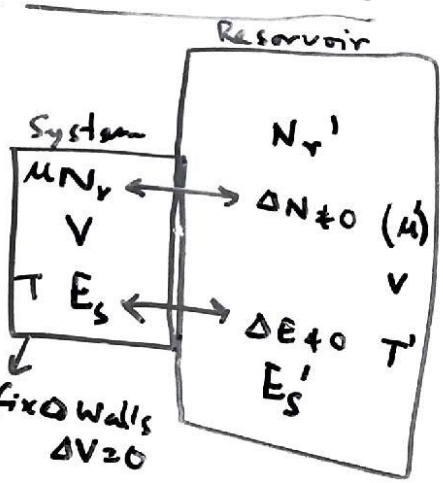
Grand Canonical Ensemble

$N_r + N_r' = N^{(0)}$

$E_s + E_s' = E^{(0)}$

$\frac{N_r}{N^{(0)}} = 1 - \frac{N_r'}{N^{(0)}} \ll 1$

$\frac{E_s}{E^{(0)}} = 1 - \frac{E_s'}{E^{(0)}} \ll 1$



$P_{r,s} \propto \Omega'(N_r', E_s')$

$\propto \Omega'(N^{(0)} - N_r, E^{(0)} - E_s)$

$\ln \Omega'(N^{(0)} - N_r, E^{(0)} - E_s)$

$= \ln \Omega'(N^{(0)}, E^{(0)}) + \left.\frac{\partial \ln \Omega'}{\partial N'}\right|_{N'=N^{(0)}} (N_r' - N^0)$

$+ \left.\frac{\partial \ln \Omega'}{\partial E'}\right|_{E'=E^{(0)}} (E_j' - E^0) + \dots$

$\left.\frac{\partial (\ln \Omega')}{\partial N'}\right| = -\frac{\mu'}{kT'}$

$\approx \ln \Omega'(N^{(0)}, E^{(0)}) + \frac{\mu'}{kT'} N_r - \frac{1}{kT'} E_s$

In eqm bet. system & Reservoir

$T' = T \quad \mu' = \mu$

$\Omega' \propto e^{-\alpha N_r - \beta E_s}$

Normalised distribution where $\alpha = -\frac{\mu}{kT}$, $\beta = \frac{1}{kT}$

$$P_{r,s} = e^{-\alpha N_r - \beta E_s} / \sum_{r,s} e^{-\alpha N_r - \beta E_s}$$