VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: Standing Waves in a Linear Bounded Medium

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Standing Waves in a Linear Bounded Medium.

When a plane progressive wave travelling along a linear bounded medium suffers reflection at the boundary, two identical waves (incident and reflected wave) i.e. of same amplitude and frequency have produced, which are travelling along the same linear path but in opposite directions. The superposition of these two waves forms standing/stationary wave. The positions of maxima and minima of the displacement remains fixed throughout. The positions where the displacements are zero (minimum) are called nodes whereas which have maximum displacements are called antinodes. The halfway between adjacent nodes are antinodes.

There are two cases :

① When reflection occurs at a rigid boundary

Let, the incident wave can be represented as, $\boxed{y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)}$. [Say, moves along +ve x-direction]

Now, this wave is incident normally on a fixed or rigid boundary and gets reflected from it.

The amplitude of the reflected wave will be negative as there is phase reversal.
The reflected wave can be represented as,

$$\boxed{y_2 = -a \sin \frac{2\pi}{\lambda}(vt+x)} \quad \left[\begin{array}{l}\text{moves along} \\ -\text{ve } x\text{-direction}\end{array}\right]$$

These two waves are moving along same linear path and superimposed.
Thus from principle of superposition, the resultant displacement

$$y = y_1 + y_2$$

or, $y = a \sin \frac{2\pi}{\lambda}(vt-x) - a \sin \frac{2\pi}{\lambda}(vt+x)$.

or, $y = -2a \cos \frac{2\pi vt}{\lambda} \sin \frac{2\pi x}{\lambda}$

or, $\boxed{y = -2a \sin \frac{2\pi x}{\lambda} \cos \omega t}$ $\quad \because \omega = \frac{2\pi v}{\lambda}$

The resultant wave (standing) is also a simple harmonic wave of same wavelength and time-period but its amplitude ~~is~~ changes
Its amplitudes is now a function of position (x

The displacement (y) is zero when

$\sin \frac{2\pi x}{\lambda} = 0$ for all values of time 't'.

or, $\frac{2\pi x}{\lambda} = m\pi$ where 'm' is an integer.

$\therefore \boxed{x = \frac{1}{2} m\lambda}$ ie. $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots$

These points of zero displacement are called <u>nodes</u>.

Again, The displacement is maximum when

$\sin \frac{2\pi x}{\lambda} = \pm 1$ for all values of 't'.

or, $\frac{2\pi x}{\lambda} = (2m+1)\frac{\pi}{2}$

$\therefore \boxed{x = (2m+1)\frac{\lambda}{4}}$ ie. $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

These points are called <u>antinodes</u>.

$\therefore$ The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$. There is always a antinode between two nodes and vice-versa.

Now, let us discuss the variation of displacement w.r.t time 't' at any particular value of 'x', The distance of particle from the fixed end.

We know, $\cos\omega t = 0$

for $\omega t = (2m+1)\frac{\pi}{2}$ where $m = 0, 1, 2 \ldots$ i.e. integer.

or, $\frac{2\pi}{T}t = (2m+1)\frac{\pi}{2}$

$$\therefore \boxed{t = (2m+1)\frac{T}{4}}$$

Therefore, the displacement will be zero at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$ and so on.

And when $\cos\omega t = \pm 1$

then $\omega t = m\pi$

or, $\frac{2\pi t}{T} = m\pi$

or, $\boxed{t = \frac{mT}{2}}$

i.e. for $t = 0, \frac{T}{2}, T, \frac{3T}{2}, 2T, \ldots$ the displacement will be maximum.

Note that, $\boxed{\frac{\partial y}{\partial t} = 2a\omega \sin\frac{2\pi x}{\lambda} \sin\omega t}$.

i.e. the particle velocity is maximum for $t = (2m+1)\frac{T}{4}$ i.e. $t = \frac{T}{4}, \frac{3T}{4}, \ldots$ where the particle displacement is zero (minimum).

Also, at $t = 0, \frac{T}{2}, T, \ldots$ all particles of the medium of the stationary wave attain their maximum displacements simultaneously whereas these particles have zero velocity at these instants.

(2) When reflection occurs at a free boundary

Let, The incident wave is

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \text{ moving along +ve } x\text{-axis}$$

As The wave reflects back from free boundary there is no reversal of phase.

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt + x) \text{ moving along -ve } x\text{-axis.}$$

Therefore, the resultant ~~wave~~ displacement

$$y = y_1 + y_2.$$

$$\text{or, } y = a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x).$$

$$\text{or, } y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}$$

$$\text{or, } \boxed{y = 2a \cos \frac{2\pi x}{\lambda} \sin \omega t}$$

This shows the resultant wave is a Stationary wave with amplitude $\left(2a \cos \frac{2\pi x}{\lambda}\right)$ which is function of 'x'.

Particle velocity $\frac{\partial y}{\partial t} = 2a\omega \cos\frac{2\pi x}{\lambda} \cos\omega t$.

Now, $\cos\left(\frac{2\pi x}{\lambda}\right) = 0$

ie. $\frac{2\pi x}{\lambda} = (2m+1)\frac{\pi}{2}$ $\qquad m \to$ integer

ie. $\boxed{x = (2m+1)\frac{\lambda}{4}}$ a, $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

So, the particle displacement will be zero (minimum) at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$ for all values of 't'.

And $\cos\left(\frac{2\pi x}{\lambda}\right) = \pm 1$

a, $\frac{2\pi x}{\lambda} = m\pi$ $\qquad m \to$ integer.

$\boxed{x = \frac{m\lambda}{2}.}$ a, $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$

$\therefore$ the particle displacement will be maximum at all times at $x = 0, \frac{\lambda}{2}, \lambda, \ldots$

Check the variation of particle velocity with 'x' for all values of 't'.

Check the variation of displacement and particle velocity with time for all values 'x'.

Hint: Use the expressions and follow the above process.