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NAAC ACCREDITED ‘A’ GRADE

Topic: Fresnel theory of Diffraction (Zone plate)

Course Title: Wave and Optics (Theory)

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Introduction:-

In this type of diffraction pattern both source screen and diffracting obstacles are stay at finite distance. The bending of light from sharp edge was first explained by Fresnel and the phenomena may be understood clearly by using **Huygens-Fresnel** principle which is-

Each point on a wavefront is a source of secondary disturbance and the secondary wavelets emanating from different points mutually interfere.

In order to appreciate the implications of this principle we consider the incidence of a plane wave on a circular hole of radius a as shown in Fig.1. In previous notes we had shown that the beam will undergo diffraction divergence and the angular spreading will be given by

$\Delta\theta \sim \lambda/2a$

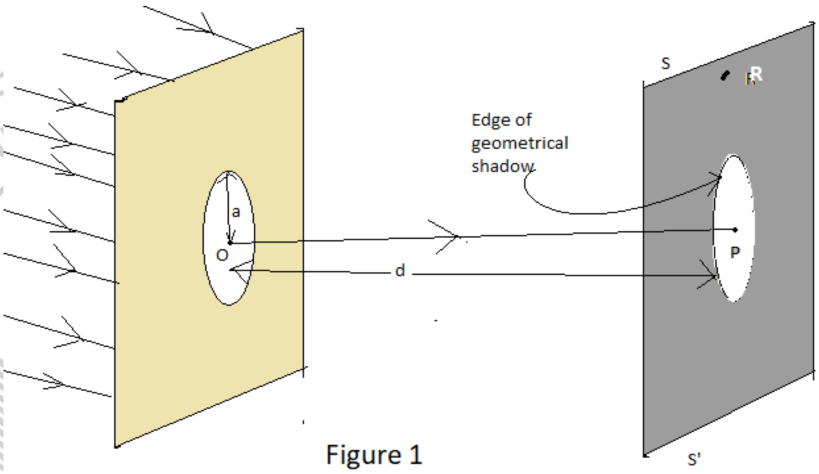


Figure 1

Thus, when a >>λ the intensity at a point R (which is deep inside the geometrical shadow) will be negligible; on the other hand, if a ~ λ there will be almost uniform spreading out of the beam resulting in an (almost) uniform illumination of the screen. This phenomenon is a manifestation of the fact that when a >>λ, the secondary wavelets emanating from different points on the circular aperture so beautifully interfere to produce (almost) zero intensity in the geometrical shadow and a large intensity inside the circular region (see Fig.1). However, if a ~ λ then the aperture almost acts as a point source resulting in a uniform illumination of the screen (see Fig. 2).

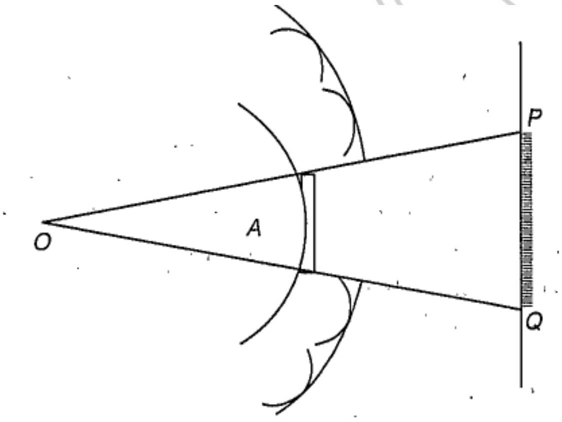


Figure 2

We will first introduce the concept of Fresnel half-period zones to have a qualitative understanding of the Fresnel diffraction pattern; this will be followed by a more rigorous analysis of the Fresnel class of diffraction and its transition to the Fraunhofer region.

FRESNEL HALF-PERIOD ZONES (Rectilinear propagation of light):-

Let us consider a plane wave front WW’ propagating in z direction as shown in fig 3.Field at an arbitrary point P due to the disturbances reaching from different portions of the wavefront, let us make the following construction: from the point P we drop a perpendicular PO on the wavefront. If PO = d, then with point P as centre we draw spheres of radii d + λ/2, d + 2λ/2, d + 3λ/2,...d + nλ/2; these spheres will intersect WW' in circles as shown in Fig. 3. The radius of the nth circle will obviously be given by

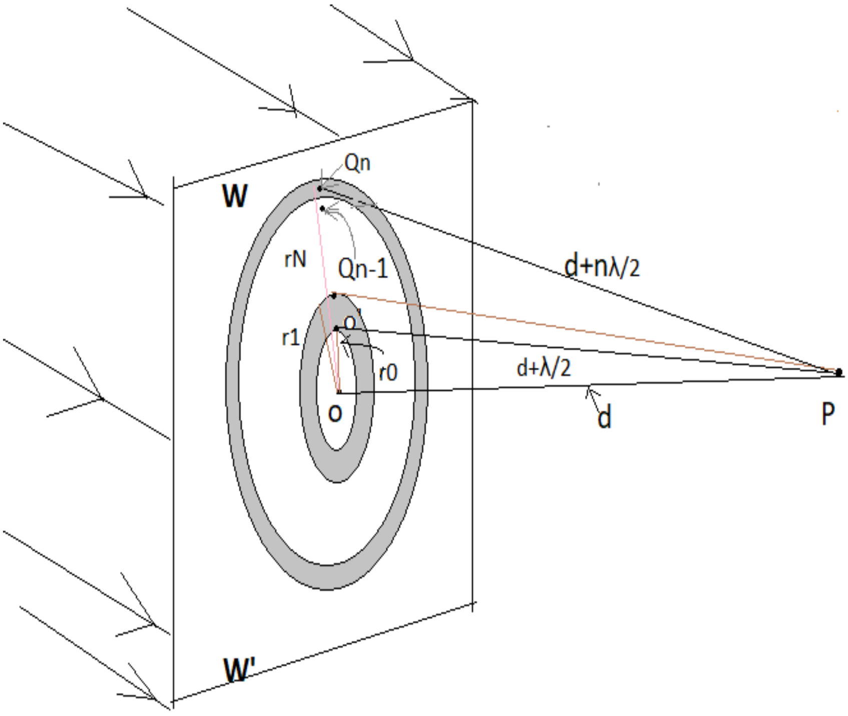


Figure 3

$$r_n = \sqrt{[\left(d + n\frac{\lambda}{2}\right)^2 - d^2]}$$

$$= \sqrt{n\lambda d}[1 + \frac{n\lambda}{4d}]^{\frac{1}{2}}$$

If $d >> \lambda$ and n is not very large (bending of light is not large in practical situation) then $n\lambda/4d << 1$ and we have $r_n = \sqrt{n\lambda d}$------------(1)

The question is why should we do that? If we observe carefully, light wave travelling from O and from O’ interfere at P in opposite phase. So intensity at P due the light from the annular area of radius r0 will diminish due to the light from 2nd annular area between the radii r0 and r1.

The annular region between nth and (n-1)th circles is known as nth half period zone and the area is given by

$A_n = \pi(r^2_n - r^2_{n-1}) = \pi[n\lambda d - (n-1)\lambda d] = \pi\lambda d$. -----------------(2)

Thus the area of each half period zone are approximately equal.

Path difference between Q_n and Q_{n-1} is λ/2 i.e. $Q_nP - Q_{n-1}P = \lambda/2$

Let us consider the intensity of light at P due to 1st zone is u1, due to 2nd zone is u2 and so on….

The resultant intensity at P is $u(P) = u1 - u2 + u3 - u4 + \ldots\ldots. + (-1)^{n+1}u_n + \ldots..$ -----------(3)

Where u_n represents the net amplitude produced by the secondary wavelets emanating from the nth zone; the alternate negative and positive signs represent the fact that the resultant disturbances produced by two consecutive zones are π out-of-phase with respect to each other. The amplitude produced by a particular zone is proportional to the area of the zone and inversely proportional to the distance of the zone from the point P; further, it also depends on an obliquity factor which is proportional to $\frac{1}{2}(1 + \cos\chi)$ where χ is the angle that the normal to the zone makes with the line QP; this obliquity factor comes out automatically from rigorous diffraction theory (in which we have no interest at that moment). Thus we may write

$$u_n = \text{constant.}\frac{\mathrm{An}}{\mathrm{QnP}}\frac{(1+\cos\chi)}{2} \text{ ----------------------------(4)}$$

Where An represents the area of the nth zone. It can be shown that if we use the exact expression for r_n , the area of the zones increase with n; however, this slight increase in the area is exactly compensated by the increased distance of the zone from the point P. In spite of this, the amplitudes u1, u2, u3... decrease monotonically because of increased obliquity. Thus we may write

$$u1 > u2 > u3 \ldots \text{ ------------------------------(5)}$$

The series expressed by Eq. (3) can be approximately summed due to a method by Schuster. We rewrite Eq. (3) as

$$u(P) = \frac{u1}{2} + \left(\frac{u1}{2} - u2 + \frac{u3}{2}\right) + \left(\frac{u3}{2} - u4 + \frac{u5}{2}\right) + \cdots \text{------------------------(6)}$$

Where the last term would either $\frac{1}{2}u_n$ or $(1/2.u_{n-1} - u_n)$according to n being odd or even. If the obliquity factor is such that

$$u_n > 1/2(u_{n-1} + u_{n+1}) \text{ --------------------------------------(7)}$$

Then the quantities inside the brackets in Eq. (6) will be negative; consequently

$$u(P) < \frac{1}{2}u1 + \frac{1}{2}u_n \text{ (n is odd) ---------------------------------(8)}$$

and

$$u(P) < \frac{1}{2}u1 + \frac{1}{2}u_{n-1} - u_n \approx \frac{u1}{2} - \frac{1}{2}u_n \text{ ; (n is even). ---------------------------(8)}$$

Where we have assumed that the amplitude of the fields produced by consecutive zones differ only slightly. In order to obtain the upper limits, we rewrite Eq. (3) in the form

$$u(P) = u1 - \frac{u2}{2} - \left(\frac{u2}{2} - u3 + \frac{u4}{2}\right) - \left(\frac{u4}{2} - u5 + \frac{u6}{2}\right) + \cdots \text{------------------------(9)}$$

Where the last term would either $-\frac{1}{2}u_n$ or $(-1/2.u_{n-1} + u_n)$ according to n being even or odd. Since the quantity inside the brackets are negative, we obtain

$$u(P) > u1 - \frac{1}{2}u2 - \frac{1}{2}u_{n-1} + u_n \approx \frac{u1}{2} + \frac{1}{2}u_n \text{ ; (n is odd). ----------------------------(10)}$$

and

$$u(P) > u1 - \frac{1}{2}u2 - \frac{1}{2}u_n \approx \frac{u1}{2} - \frac{1}{2}u_n \text{ ; (n is even). ----------------------------(10)}$$

using equation 8 and 10 we may write approximately

$$u(P) \approx \frac{u1}{2} - \frac{1}{2}u_n \text{ ; (when n is even).}$$

$$u(P) \approx \frac{u1}{2} + \frac{1}{2}u_n \text{ ; (when n is odd).}$$

For large value of n u_n can be neglected and then we have

$$u(P) \approx u_1/2 \text{ ----------------------------------(11)}$$

Implying that the resultant amplitude produced by the entire wavefront is only one half of the amplitude produced by the first half-period zone.

Intensity I ∝ $u_1^2/4$

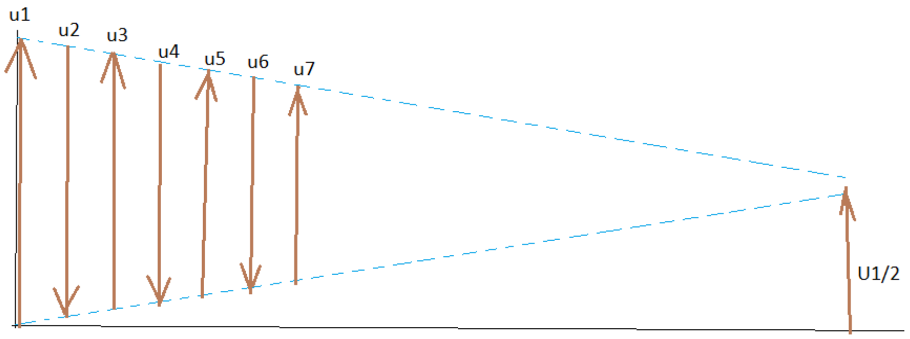


figure 4

Thus, the intensity at P is only one-fourth of that due to the first half period zone alone. Here, only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the area of the first half period zone placed at O will screen the effect of the whole wavefront and the intensity at P due to the rest of the wavefront will be zero. While considering the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round comers (diffraction effects) cannot be noticed. In the case of sound waves, the wavelengths are far greater than the wavelength of light, and hence the area of the first half period zone for a plane wavefront of sound is very large. If the effect of sound at a point beyond an obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect. If the size of the obstacles placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also.

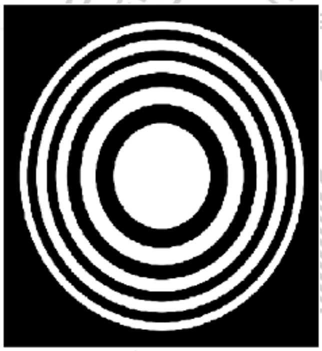
Thus, rectilinear propagation of light is only approximately true.

ZONE PLATE:

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method in dividing a wavefront into half period zone can be verified with the help of zone plate.

To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers ($r_n = \sqrt{n\lambda d}$). The odd numbered zones (i.e. 1st, 3rd, 5th, etc) are covered with black ink and a reduced photograph is taken. The drawing appears as Zone Plate shown in Fig. 5 (a). The negative of the Fig.5 (a) photograph will be as shown in Fig .5(b). In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

figure 5 (a)



b

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say b cm from the zone plate (Fig. 6) XO is the upper half of the incident plane wavefront. P is the point at which the

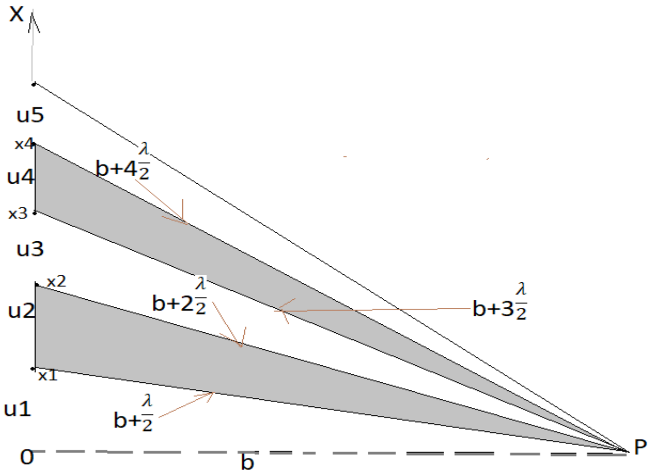


figure 6

light intensity is to be considered. The distance of the point P from the wavefront is b. o-x1(r1), o-x2(r2), o-x3(r3) are the radii of the zones.

Fig. 6 r1 = $\sqrt{\lambda b}$ and r2 = $\sqrt{2\lambda b}$ where λ is the wavelength of light. So $r_n = \sqrt{n\lambda b}$ or

$b = r_n^2 / n\lambda$ ----------------------------------(12)

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wavefront can be taken as a plane one with respect to the small area of the zone plate. The even numbered zones cut off the light and hence resultant amplitude at P is A= u1 + u3 + u5 +…… etc. In this case the focal length of the zone plate F_n, is given

$F_n = b = r_n^2 / n\lambda$ -----------------------------(13)

Thus, a zone plate has different foci for different wavelengths. The radius of the nth zone increases with increasing value of a. It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than that when the whole wavefront is exposed to the point P.

In the first case the resultant amplitude is given by

A = u1 + u3 + u5 + u7 + ………… (n is odd)

When the whole wavefront is unobstructed, the amplitude is given by

A=u1 - u2 + u3 - u4 + u5 - u6 + u7 +……….≈ u1/2 (if n is very large and n is odd).

If a parallel beam of white light is incident on the zone plate, different colours come to focus at different points along the line OP. Thus, the function of a zone plate is similar to that of a convex lens a formula for the relation between object and image may be obtained for zone plate also.

Calculation of focal length of a zone plate:-

The zone-plate can also be used for imaging points on the axis, e.g., if we have a point source at S then a bright image will be formed at P, where the point P should be such that

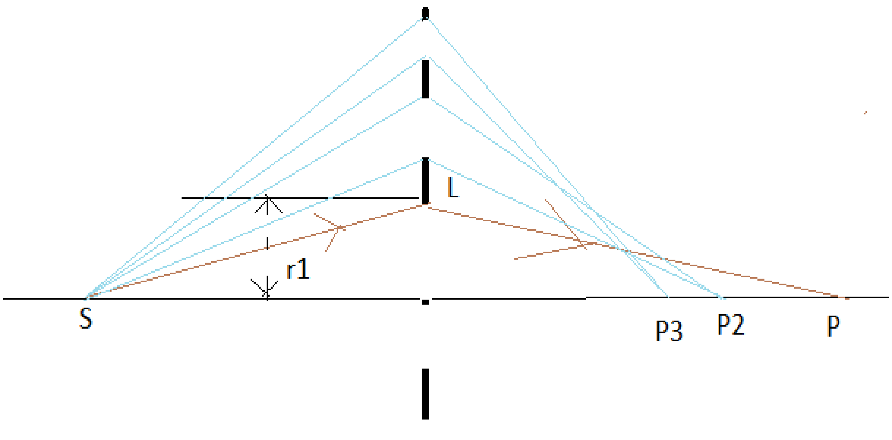


Figure 7

$\mathrm{SL} + \mathrm{LP} - \mathrm{SP} = \lambda/2$ -------------------------(14)

The point L being on the periphery of the first circle of the zone plate [see Fig. 7]. If the radius of the first circle is r1, then

$\mathrm{SL} + \mathrm{LP} - \mathrm{SP} = \sqrt{(a^2 + r1^2)} + \sqrt{(b^2 + r1^2)} - (a + b)$

$\approx a[1 + \frac{r1^2}{2a^2}] + b[1 + \frac{r1^2}{2b^2}] - (a + b) \approx \frac{r1^2}{2}(\frac{1}{a} + \frac{1}{b})$ ----------------------(15)

Thus eq (14) become

$\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{f}$ ------------------------------(16)

Where $f = r1^2/\lambda$ represents the focal length. Equation (16) resembles the lens law.

A very simple practical application of zone plate

Mobile screen magnifier

Comparison between a zone plate and a convex lens:

1. Both the zone plate and convex lens form real image of an object and the equations of conjugate distances are similar.
2. The focal lengths of both depend on the wavelength, and hence suffer from chromatic aberration. The chromatic aberration in a zone plate is much more severe than in a convex lens.
3. A zone plate acts simultaneously as a convex lens and as a concave lens. In addition to a real image, a virtual image is also formed simultaneously. A convex lens forms only a real image.
4. In case of zone plate the image is formed by the diffraction phenomenon.

In case of a convex lens the image is formed due to refraction of light.

5. The zone plate has got multiple foci on either side of the plate. Hence, the intensity of the image formed will be much less. Convex lens has only one focus. As all the light is focused at one point, the intensity of the image will be more.

6. In a zone plate, waves reaching the image point through any two alternate zones differ in path by λ and in phase by 2π . In case of a convex lens all the rays reaching the image point have zero path or phase difference.
7. A zone plate can be used over a wide range of wavelengths from microwaves to x-rays. Glass lens cannot be used beyond the visible region.

DISTINCTION BETWEEN INTERFERENCE AND DIFFRACTION

The main differences between interference and diffraction are as follows:

INTERFERENCE	DIFFRACTION
1. Interference is the result of interaction of light coming from different wave fronts originating from the source.	1. Diffraction is the result of interaction of light coming from different parts of the same wavefront.
2. Interference fringes may or may not be of the same width.	2. Diffraction fringes are not of the same width.
3. Regions of minimum intensity are perfectly dark.	3. Regions of minimum intensity are not perfectly dark.
4. All bright bands are of same intensity.	4. The different maxima are of varying intensities with maximum intensity for central maximum.

Diffraction by a circular aperture using concept of half period zone:

We may use the above analysis to study the diffraction of a plane wave by a circular aperture. Let the point P be at a distance d from the circular aperture (see Fig. 1). We assume that the radius of the circular aperture ‘a’ can be increased from zero onwards. As a increases, the intensity at the point P would also increase till the circular aperture contains the first half-period zone; this would happen when a $=\sqrt{(\lambda d)}$. The resultant amplitude at the point P would be u_1 which is twice the value of the amplitude for the unobstructed wavefront [see Eq.(11)]. The intensity would therefore be $4I_0$, where I_0 represents the intensity at the point P due to the unobstructed wavefront. If we further increase ‘a’ then u(P) would start decreasing and when the circular aperture contains the first two half-period zones (which would happen when $a = \sqrt{(2d\lambda)}$ the resultant amplitude (= u1 - u2) would be almost zero. Thus, by increasing the hole diameter, the intensity at the point P decreases almost to zero. This interesting result is once again due to the validity of the Huygens-

Fresnel principle and hence would be valid for sound waves also. We may generalize the above result by noting that if the aperture will contain an odd number of half-period zones and the intensity will be maximum;

$a = \sqrt{(2n+1)\lambda d}$; n= 0,1,2…..[maxima]

On the other hand, if the aperture will contain an even number of half-period zones and the intensity will be minimum.

$a = \sqrt{2n\lambda d}$; n= 0,1,2…..[minima]

As a corollary of the above analysis we can consider a circular aperture of a fixed radius a and study the intensity variation along the axis. Whenever the distance

$d = \frac{a^2}{(2n+1)\lambda}$; n=0, 1, 2….. (maxima) the point P (see Fig.1) will correspond to a maximum.

Similarly, when

$d = \frac{a^2}{2n\lambda}$; n = 1, 2 ,... (minima) the point P will correspond to a minimum.

The intensity distribution on a screen SS' at off-axis points can be approximately calculated by using the half-period zones, but such a calculation is fairly cumbersome. However, from the symmetry of the problem, one can deduce that the diffraction pattern has to be in the form of concentric circular rings with their centres at the point P.