VIVEKANANDA COLLEGE

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KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: Resolving Power

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1. RESOLVING POWER

When two objects or their images are very close to each other, they appear as one and it may not be possible for the eye to see them as separate. If the objects are not seen separately, then we say that the details are not resolved by the eye. Optical instruments are used to assist the eye in resolving the objects or images. The method adapted to seeing the close objects as separate objects is called **resolution**. *The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power*.

We use the term 'resolving power' in two different senses. In case of microscopes and telescopes, we talk of geometrical resolution where the geometrical positions between two nearby objects are to be resolved and in case of spectroscopes we refer to spectral resolution where differences of wavelengths of light in a given source are to be resolved.

Definition

Resolving power is normally defined as the reciprocal of the smallest angle subtended at the objective of optical instrument by two point objects, which can just be distinguished as separate.

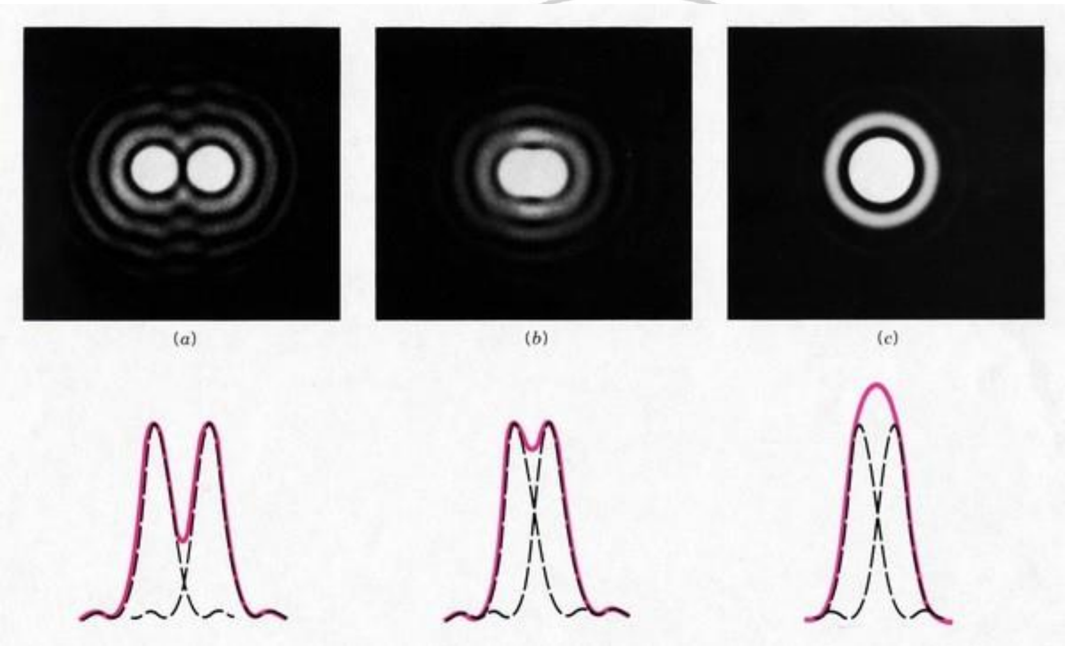
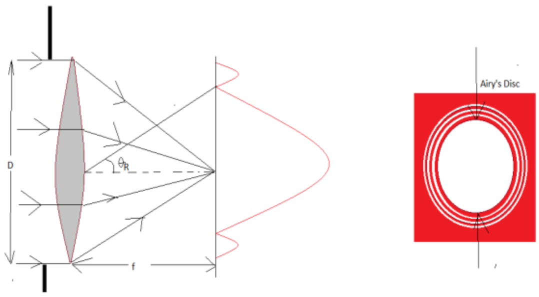


Figure 1

in the figure (a) represent fully resolved images,(b) is just resolved images, and (c) showing the images are not resolved.

2. RAYLEIGH'S CRITERION

When a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is a bright surrounded by alternate dark and bright rings (see Fig .2).



It is known as Airy's disc. If there are two point objects lying close to each other, then two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them as separate (see Fig. 1).

To obtain the measure of the resolving power of an objective lens Rayleigh suggested that the two images of such point-objects lying close to each other may be regarded as separated ***if the central maximum of one falls on the first minimum of the other. In other words, when the central bright image of one falls on the first dark ring of the other, the two images are said to be just resolved*** (see Fig. 1 b and fig 3). This is equivalent to the condition that the distance between the centers of the patterns shall be equal to the radius of the central disc. This is called the **Rayleigh criterion** for resolution and is also known as **Rayleigh's limit of resolution**

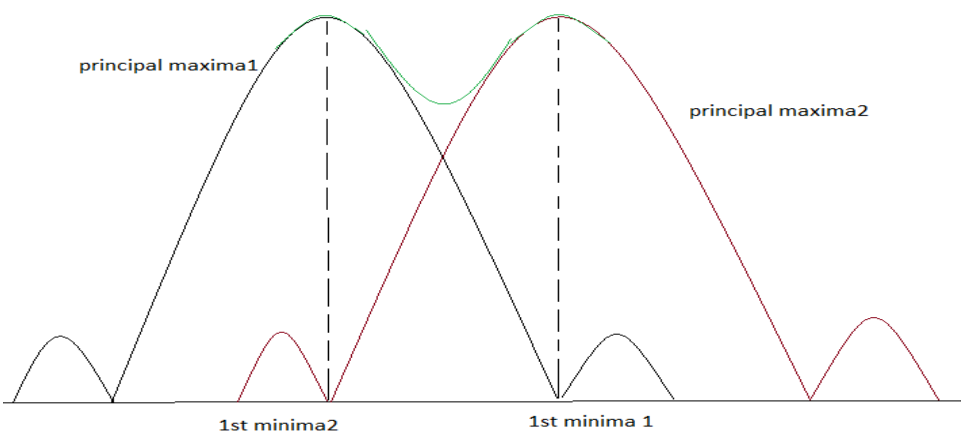


Figure 3 condition for just resolve from Rayleigh's criterion

3. **LIMIT OF RESOLUTION OF THE EYE**

In Fig. 4, MN is the eye lens, A and B are two object points separated by a distance h and A' and B' are the corresponding image points at a distance h' formed on the retina. μ is the refractive index of the object medium and μ' is the refractive index of the image medium. If the object is placed in air, μ = 1 and the image medium is vitreous humor whose refractive index is 1.33. If the object is situated at the least distance of distinct vision, u = 25 cm = 250 mm for a normal eye. If the diameter of the eye ball is about 2.5 cm, then v = 2.5 cm = 25 mm approximately. Taking the pupillary diameter of the eye as 2 mm, R = 1 mm.

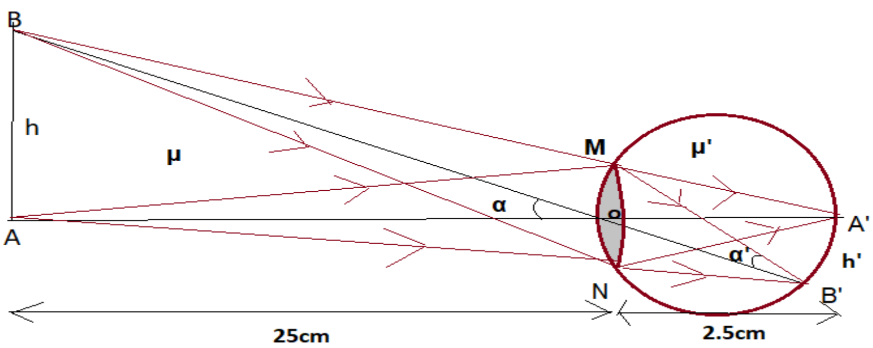


Fig 4

Also, human eye is most sensitive to a wavelength λ_0 = 5500 Å.

From the ΔAMO for small angles of θ,

$$\sin\theta = \tan\theta = \frac{R}{u} = \frac{1}{250} = 0.004$$

Numerical aperture = μ sinθ = 0.004

Applying Rayleigh's criterion, the minimum distance (h) between two just resolvable object points of equal intensity is given by,

$$h = \frac{0.61\lambda 0}{\mu\, R \sin\theta} = \frac{0.61\times 5500\times 10^{-8}}{0.004} = \frac{1}{100}\, cm \text{ approximately.}$$

It means that if the object is situated at the least distance of distinct vision from the eye (25 cm), the minimum separation between two nearby object points should be of the order of 0.1 mm. If the object points are separated by a distance larger than 0.1 mm, they are clearly visible and are well resolved.

Similarly, the distance h' between the centers of the two images is given by,

$$h' = \frac{0.61\lambda 0}{\mu\prime\, R \sin\theta\prime} = \frac{0.61\times 5500\times 10^{-8}}{1.33\times 0.04} = \frac{1}{1000}\, cm$$

Also, $\alpha = \sin\alpha = \frac{0.61\lambda 0}{\mu\, R} = \frac{0.61\times 5500\times 10^{-8}}{1\times 0.1}$ = 0.00034 radian = 1 minute of an arc (approx.)

The value of h' (= **10^{-2}** mm) is approximately equal to the distance between the cones fovea and thus the retinal structure is strikingly in accordance with the limit of resolution of eye. Further, two point objects appear to be just resolved if the angle subtended by them at the minute of an arc. If the diameter of the pupil of the eye is smaller than 2 mm, the numerical aperture decrease and hence the value of h increases, i.e. two points will appear to be just resolved if the distance between the two is larger. Thus the resolving ability of the eye is decreased.

LIMIT OF RESOLUTION OF A CONVEX LENS

In Fig.5 L is a convex lens. A and B are two object points and A‘and B' are the corresponding points. The distance between the object points is h and the distance between the image point is h’.The distance of the object points from the lens is u and the distance of the image points is v. μ are the refractive indices of the object and image media.

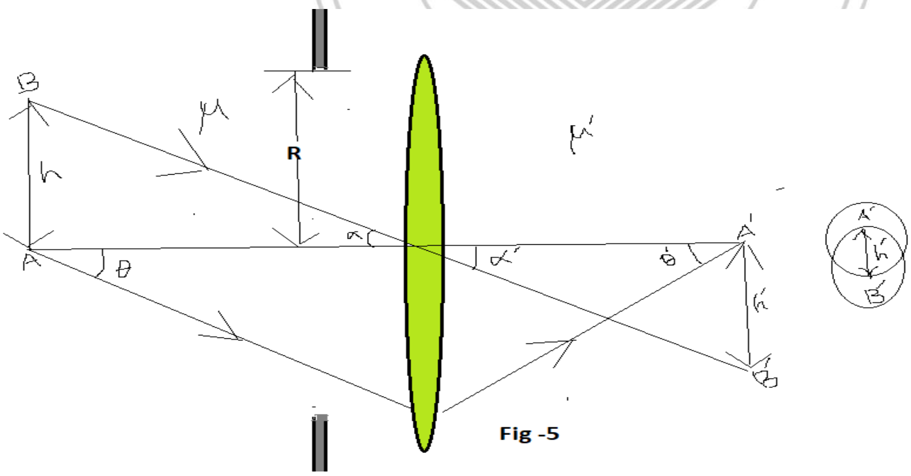


Fig -5

R is the radius of the aperture kept in front of the lens (D is the diameter of the aperture). In the side figure, A' and B' are the centers of the central bright discs of the patterns of A and B. Let λ and λ' be the wavelengths of light in the object and image media and λ_0, be the wavelength of light in vacuum. Then,

$\lambda = \frac{\lambda 0}{\mu}$ and $\lambda' = \frac{\lambda 0}{\mu\prime}$

According to Rayleigh criterion, if the two images are just resolved, distance between the centers of the two discs (h’) is equal to the radius of either disc. If this condition is satisfied, then,

$$\sin\alpha = \frac{1.22\lambda}{D} = \frac{1.22\lambda0}{\mu D} = \frac{1.22\lambda0}{2\mu R} = \frac{0.61\lambda0}{\mu R} \text{ --------------------(1)}$$

Similarly,

$$\sin\alpha' = \frac{0.61\lambda0}{\mu\prime R} \text{ -----------------------(2)}$$

when the angles α and θ are small, we write

$\sin\alpha = \tan\alpha = h/u$

and

$\sin\theta = \tan\theta = R/u$ or $R = u\sin\theta$

Substituting the values of sin a and R in equation (1),

$\mu u \sin\theta . h/u = 0.61\lambda_0$

$h = 0.61\lambda_0 / \mu \sin\theta$

Similarly it can be shown that,

$h' = 0.61\lambda_0 / \mu' \sin\theta'$

Thus, according to the Rayleigh's criterion of resolution, the linear distance between two just solved point objects is given by,

$h = 0.61\lambda_0 / \mu \sin\theta$

and the distance between corresponding image points is given by

$h' = 0.61\lambda_0 / \mu' \sin\theta'$

the quantity (μ sinθ) is called numerical aperture(N.A) of the optical instrument.

Resolving power of optical instrument:-

To express the resolving power of an optical instrument as a numerical value, Lord Rayleigh proposed an arbitrary criterion. According to him two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice-versa. The same criterion can be conveniently applied to calculate the resolving power of a telescope, microscope, grating, prism, etc.

In Fig-6, A and B are the central maxima of the diffraction patterns of two spectral lines of wavelength λ1 and λ2. The difference in the angle of diffraction is large and two images can be seen as separate ones. The angle of diffraction is greater than the angle of diffraction corresponding to the first minimum at the sight of A. Hence the two spectral lines will appear well resolved.

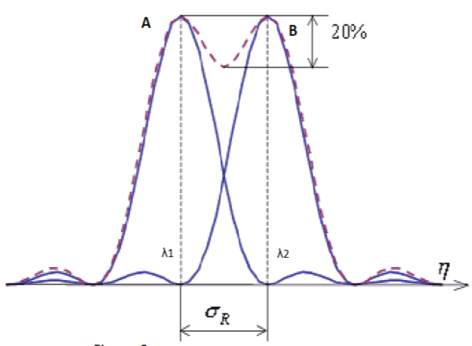


Figure 6

RESOLVING POWER OF PRISM

Resolving power signifies the ability of the instrument to form separate images of two neighbouring wavelengths λ and $\lambda+d\lambda$ in the wavelength region λ.

In the given Figure 7, S is a source of light, L1 is a collimating lens and L2 is the telescopic objective lens. As the two wavelengths λ and $\lambda+d\lambda$ are very close ,if the prism is in minimum deviation position it would hold good for both the wavelengths.

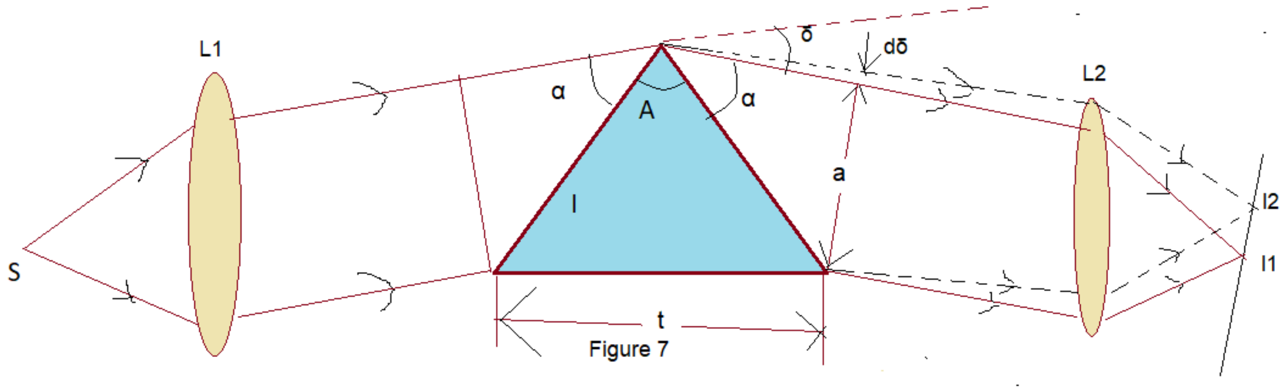


Figure 7

The final image I1 corresponds to the principal maximum for wavelength λ and image I2 corresponds to the principal maxima for wavelength $\lambda + d\lambda$. I1 and I2 are formed at the focal plane of the telescopic objective L2.

In the case of diffraction at a rectangular aperture the position of I2 will correspond to the first maximum of image I1 for wavelength λ 1 provided

$a(d\delta) = \lambda$ or $d\delta = \lambda / a$ -------------------------------------- (3)

Here δ is the angle of minimum deviation for wavelength λ

From Figure 7 $\alpha + A + \alpha + \delta = \pi$

$\Rightarrow \alpha = [(\pi/2) - (A + \delta)/2]$

Therefore, $\sin \alpha = \sin [(\pi/2) - (A + \delta)/2]$

or $\sin\alpha = \cos [(A + \delta)/2]$

But $\sin\alpha = a/l$

Therefore $\cos [(A +\delta)/2] = a/l$ ------------------------------------- (4)

Also $\sin [A/2]=t/2l$ ------------------------------------- (5)

In case of prism

$\mu = \sin [(A + \delta)/2]/\sin [A/2]$

Therefore $\sin [(A + \delta)/2] = \mu \sin [A/2]$ ------------------------------------- (6)

Here μ and δ are dependent on wavelength of light λ.

Differentiating equation (6) with respect to λ. We get

$$\frac{1}{2}\cos\left(\frac{A+\delta}{2}\right)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}\left(\sin\frac{A}{2}\right)$$

Using equations (4) and (5)

$$\frac{1}{2}(a/l)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}(t/2l)$$

Therefore

$$a\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}t$$

Substituting the value in equation 3

$\frac{\lambda}{d\lambda} = \frac{d\mu}{d\lambda}t$----------------------------(7)

The expression λ/dλ measures the resolving power of the prism.

It is defined as the ratio of the wavelength λ to the smallest difference in wavelength dλ, between this line and a neighbouring line such that the two lines appear just resolved, according to Rayleigh's criterion.

So, resolving power of a prism = $\frac{d\mu}{d\lambda}t$

It means that the resolving power (i) is directly proportional to the length of the base of the prism and (ii) rate of change of refractive index with respect to wavelength for that particular material.

RESOLVING POWER OF MICROSCOPE

In the case of microscope the object is very near the objective of the microscope and the objects subtend very large angle at the objective. The limit of resolution of a microscope is determined by the least permissible linear distance between the two objects so that the two images are just resolved.

The minimum distance by which two points in the object are separated from each other so that their images as produced by the microscope are just seen separate is called the limit of resolution. The reciprocal of limit of resolution is known as the resolving power.

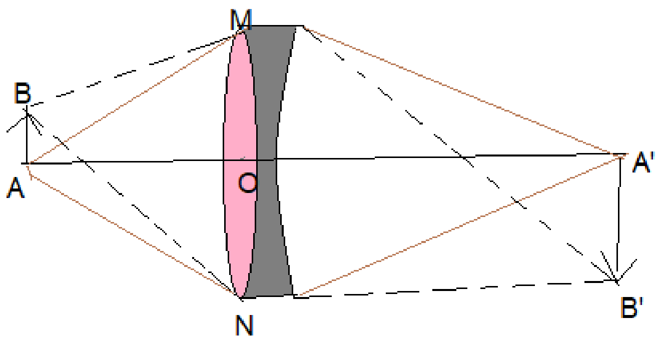


Figure 8

In Fig. 8, MN is the aperture of the objective of a microscope and A and B are two object points at a distance d apart. A' and B' correspond to diffraction patterns due to A and B. A' and B' are surrounded by alternate dark and bright diffraction rings. The two images are said to be just resolved if the position of the central maximum of B' also corresponds to the first minimum of the image of A'.

The path difference between the extreme rays from the point B and reaching A' is given by,

(BN+ NA') - (BM+MA') , But NA'=MA'

Therefore, path difference = BN-BM

In the figure 9, AD is perpendicular to DM and AC is perpendicular to BN and AB=d.

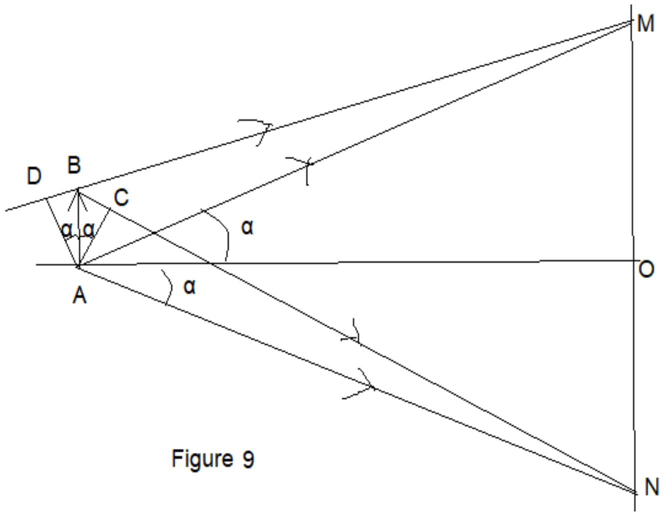


Figure 9

Therefore,

BN-BM=(BC+CN)-(DM-DB)

But, CN=AN=AM=DM

Therefore, Path Difference=BC+DB

From triangles ACB and ADB

BC = AB sinα = d sinα

and DB = AB sinα = d sinα

Path Difference = 2d sinα

If this path difference 2d sinα=1.22λ then A' corresponds to the first minimum of image B' and two images appear just resolved.

Therefore,

2d sinα = 1.22λ

or $\boldsymbol{d} = \frac{\mathbf{1.22}\boldsymbol{\lambda}}{\mathbf{2sin\ \alpha}}$ -----------------------------------(8)

Thus above equations give the resolving power of microscope.

RESOLVING POWER OF TELESCOPE

Let 'd' be the diameter of the objective of the telescope considering the incident ray of light from two neighbouring points of a distant object (in fig 10). The image of each point object is a Fraunhofer diffraction pattern.

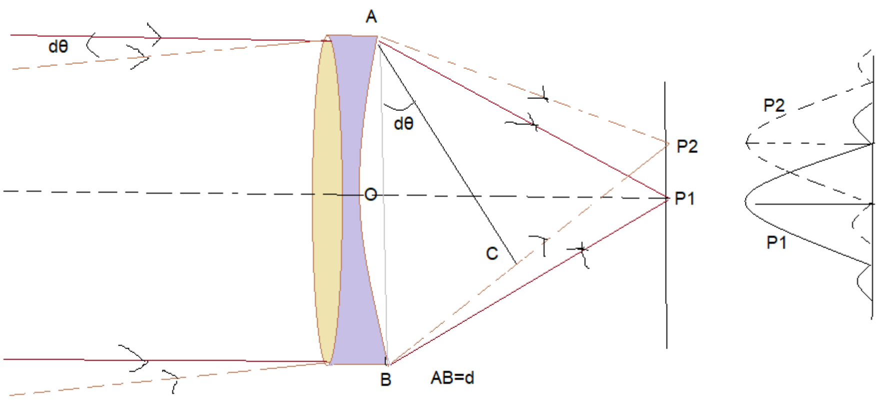


Figure 10

Let P1 and P2 be the position of the central maximum of two images. These two images are resolved if the position of central maximum of second image coincides with the first maximum of the first image and vice-versa. The path difference between the second wave traveling in the directions AP1 and BP1 is zero and hence they reinforce with one another at P1.

The secondary waves traveling in the directions AP2 and BP2 will meet at P2 on the screen. Let the angle P_2AP_1 be dθ. The path difference between the secondary waves traveling in the directions BP2 and AP2 is equal to BC.

Therefore,

$BC = AB \sin d\theta = AB\, d\theta = d.\, d\theta$ (for small angle)

If this path difference [$d.\, d\theta = \lambda$], the position of P2 corresponds to the first minimum of the first image. But P2 is also the position of the central maximum of the second image. Thus Rayleigh's condition of resolution is satisfied if,

$d.d\theta = \lambda$ or $d\theta = \lambda/d$. -------------------------(9)

The whole aperture AB can be considered to be made of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\lambda/2$. All the secondary waves destructively interfere with one another and hence p2 will be the first minimum of the first image. The equation 9 holds good for rectangular aperture. For circular aperture this equation can be written as

$d\theta = 1.22\lambda/d$

The reciprocal of dθ measures the resolving power of the telescope.

$1/d\theta = d/(1.22\lambda)$ ---------------------------(10)

dθ is also the angle subtended by the two distant object points at the objective. From the equation 10 , it is clear that a telescope with large diameter of the objective has higher resolving power.

RESOLVING POWER OF PLANE DIFFRACTION GRATING

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighbouring line such that two lines appear to be just resolved.

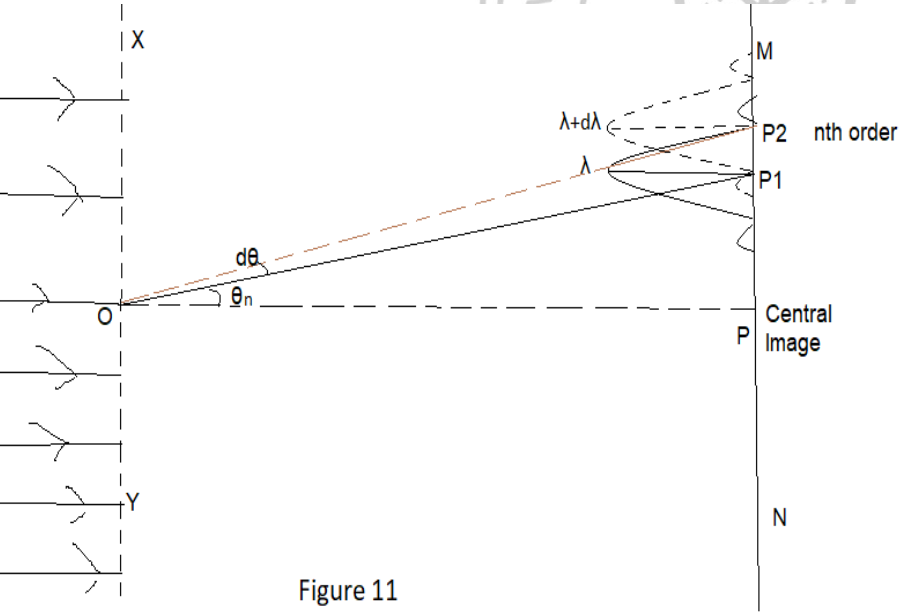


Figure 11

In above figure, XY is a grating surface and MN is the field of view of the telescope.P1 is the nth primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n . P2 is the nth primary maximum of a second spectral line of wavelength λ+dλ at a diffracting angle of $\theta_n + d\theta$. P1 and P2 are the spectral lines in the nth order.

The direction of nth primary maximum for a wavelength λ is given by

$(a + b) \sin \theta n = n\lambda$ --------------------- (11)

The direction of nth primary maximum for a wavelength (λ+dλ) is given by

$(a + b) \sin (\theta n+d\theta) = n (\lambda+d\lambda)$ ------------------------------------ (12)

These two lines will appear just resolved if the angle of diffraction ($\theta n + d\theta$) also corresponds to the direction of the first secondary minimum after the nth primary maximum at P1.

This is possible if the extra path difference introduced is λ/N where N is the total number of lines of the grating surface.

Therefore

$(a + b) \sin(\theta n + d\theta) = n\lambda + \lambda/N$ ------------------------------------- (13)

Equating the right hand sides of (12) and (13)

$n(\lambda + d\lambda) = n\lambda + \lambda/N$ or $nd\lambda = \lambda/N$ or $(\lambda/d\lambda) = nN$

The quantity $\lambda/d\lambda = nN$ measures the resolving power of a grating.

Thus, the resolving power of a grating is independent of the grating constant. The resolving power is directly proportional to (i) the order of the spectrum and (ii) the total number of lines on the grating surface. For a given grating, the distance between the spectral lines is double in the second order spectrum than that in the first order spectrum.

High dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to show nearby spectral lines as separate ones.

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