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NAAC ACCREDITED ‘A’ GRADE

Topic: Dielectric Materials- Class-4

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Dielectrics

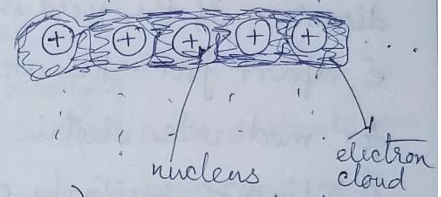
Matter can be classified into two categories depending on how they respond to electrostatic fields.

① Conductors

② Insulators or dielectrics

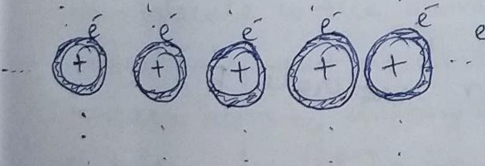
Conductors

Plenty supply of free electrons (charge carriers) through out the material. These free electrons can move freely inside a conductor but cannot leave the conductor surface because of surface barrier potential. They can move this way because they are not bound/attached to any particular atom or nucleus. For typical ~~meta~~ metals no. of such electrons is one or two per atom.



Dielectrics

In dielectrics all electrons (i.e, charges) are attached to specific atoms or molecules. These electrons are strongly bound to the corresponding atoms and hence can't move freely as in case of conductor. But they can make microscopic displacements from their equilibrium position in presence of an electric field. But this displacement happens within the atom and the displacements of all electrons inside the corresponding atoms as a whole makes a cumulative effects which results in the characteristic behaviour of dielectric materials.



e⁻s are in their shells inside the atoms i.e, attached to particular atoms

An applied electric field can distort the charge distribution of such atoms of dielectric materials in two mechanisms – ① stretching ② Rotating.

②

Effect of electric field : Induced dipoles

As we know, an atom is electrically neutral. Apparently, when an electric field is applied to an atom one may expect that the field will not influence the neutral atom. But this is not the case. Even if the atom is electrically neutral as a whole, it has a positively charged nucleus in the core and a negatively charged electron cloud surrounding the core. These two regions of charge inside an atom are influenced separately by the electric field. The nucleus is pushed slightly in the direction of the field and the electrons in opposite direction. If the field is sufficiently high that it can pull the $\bar{e}$ apart from the atom i.e, it can ionize the atom. However, for moderate electric field the electron is not pulled out but shifting of positively charged core and negatively charged electron cloud happens and an equilibrium is attained soon between the attractive force of nucleus and $\bar{e}$ cloud (whose centres do not coincide now) and the electric field which is pushing nucleus and electron cloud away from each other. And the atom holds itself as it was but it is now polarized as plus charge (nucleus) and negative charge ($\bar{e}$ cloud) shifted away from each other. Hence the atom has a tiny dipole moment now which is ~~directed~~ is directed along the electric field. For moderate electric field the dipole moment induced by the electric field is proportional to the field.

$$\vec{p} \propto \vec{E} \qquad (\text{for moderate } \vec{E})$$

$$\boxed{\vec{p} = \alpha \vec{E}}$$

α is constant of proportionality known as atomic polarizability. Its value depends on the atomic structure. For atoms the polarization is almost same for all directions of applied electric field. However, for molecules the polarization depends on direction of applied field. For example, CO_2

is a linear molecule and has different polarization for fields applied along and perpendicular to the axis of the molecules. In such molecules the polarizability has different value for different direction of applied field. And for any random direction of field with respect to the axis it has to be resolved into parallel and perpendicular components like

$$\boxed{\vec{p} = \alpha_{\perp} \vec{E}_{\perp} + \alpha_{\parallel} \vec{E}_{\parallel}}$$

In such cases the dipole moment may not be in the direction of the electric field.

For nonlinear molecule i.e, asymmetric molecule the field has to be resolved into three mutually perpendicular directions (x, y, z) and the polarizability is a tensor of rank 2. The choice of axes could be along the principal axes so that the off-diagonal terms vanish leaving only three nonzero components.

Polar molecule

The neutral atom as no dipole moment in absence of electric field. But some molecules such as water has a nonzero dipole moment in absence of electric field. This is intrinsic dipole moment arises because of the structure of the molecules and electrons tend to accumulate near any particular atom (element). Such molecules which have permanent (intrinsic) dipole moment are called polar molecules.

Polarization

So far we have studied the effect of electric fields on an individual atom or molecule in the above sections. If we place a dielectric material consisting of neutral atoms then the field induces a dipole moment in each atom by shifting mechanism. If the dielectric material is made of polar molecules having permanent dipole moments then as we know, the field will create a torque on each dipole which tries to rotate the dipole moment along the electric field. So in this case the dipole moments align along the field by rotation mechanism and results in a net dipole moment along the direction of applied field. As soon as the field is withdrawn these dipoles orient themselves in random direction due to thermal motions and hence net dipole moment becomes zero.

In both cases whether the ~~d~~ dielectric material is made of neutral atoms or polar molecules, the final result on application of electric field is same. A net dipole moment along the direction of the electric field arises inside the material. Hence the material becomes polarized.

This effect is presented by polarization $\vec{P}$ which is defined by dipole moment per unit volume.

$$\boxed{\vec{P} \equiv \text{dipole moment per unit volume.}}$$

Polarization $\vec{P}$ is a vector quantity.

The polarization appears due to electric field. And after a polarization (dipole moments) sets in it also creats an electric field. And hence the total electric field inside a material will be different from applied field.

* In polar molecules, in addition to rotation stretching or shifting also may occur. But rotation of a molecule is easier than stretching a molecule and hence the first mechanism dominates over the second one.

* In some material even after the field is withdrawn the dipole moments remain locked in the oriented state and hence gives a ~~permanent~~ ~~dipole moment~~ polarization even in absence of the electric field. Such materials are called ferroelectric material.

Field of a polarized object

When a material is polarized it has a lot of microscopic dipoles aligned in one direction. The dipole moment per unit volume is polarization $\vec{P}$. There will be an electric field due to these dipoles in addition to the external applied electric field.

We know, the potential at a point P at distance r due to a dipole having dipole moment $\vec{p}$ is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}\cdot\vec{p}}{r^2}$$

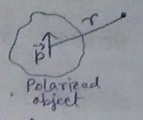
For the polarized material having polarization $\vec{P}$, the dipole moment of a volume element $d\tau'$ is

$$\vec{p} = \vec{P}\, d\tau'$$

⑤

So, the potential at the [redacted] point at distance r of the volume element $d\tau$ is

$$dV(\vec{r}) = \frac{1}{4\pi\epsilon_0}$$ [redacted] $$\frac{\hat{r}\cdot\vec{P}(r')}{r^2}\,d\tau'$$



The total potential due to the whole volume of the polarized body is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{v} \frac{\hat{r}\cdot\vec{P}}{r^2}\,d\tau'$$

$v \to$ volume
$V \to$ Potential

Now we know that, $\vec{\nabla}'(\frac{1}{r})$ [redacted] $= \frac{\hat{r}}{r^2}$

$r' \to$ source coord.
$\vec{P} = \vec{P}(r')$
[redacted]

$$\therefore\ V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{v} \vec{P}\cdot\vec{\nabla}'(\tfrac{1}{r})\,d\tau'$$

Integrating by parts,

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{v} \vec{\nabla}'\cdot\left(\frac{\vec{P}}{r}\right)d\tau' - \int_{v} \frac{1}{r}(\vec{\nabla}'\cdot\vec{P})\,d\tau'\right]$$

from divergence theorem

$$V = \frac{1}{4\pi\epsilon_0}\oint_{S} \frac{1}{r}\vec{P}\cdot d\vec{a}' - \frac{1}{4\pi\epsilon_0}\int_{v} \frac{1}{r}(\vec{\nabla}'\cdot\vec{P})\,d\tau'$$

The first term is the potential for a surface charge with surface charge density $\boxed{\sigma_b \equiv \vec{P}\cdot\hat{n}}$

where $\hat{n}$ is normal unit vector to the surface

and the second term is potential of a volume charge with volume charge density $\boxed{\rho_b = -\vec{\nabla}\cdot\vec{P}}$

This means that the potential and hence the electric field produced by a polarized object is the addition of the potential produced by a surface charge with density $\sigma_b = \vec{P}\cdot\hat{n}$ and a volume charge having density $\rho_b = -\vec{\nabla}\cdot\vec{P}$

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0}\oint_{S} \frac{\sigma_b}{r}\,da' + \frac{1}{4\pi\epsilon_0}\int_{v} \frac{\rho_b}{r}\,d\tau'}$$

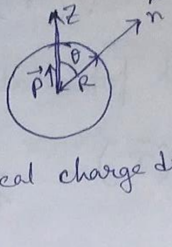
These two charge densities are bound charge densities. If we can get these two quantities of a polarized material we can calculate the potential and field of the polarized material.

Example

For uniformly polarized sphere of radius R

Let the direction of polarization of the sphere is along z axis.

The volume charge density ρ_b is zero as polarization $\vec{P}$ is uniform.



And surface bound charge density is

$$\sigma_b = \vec{P}.\hat{n} = P\cos\theta$$

where θ is the spherical coordinate.

So the field will be as produced by a spherical charge distribution of density $P\cos\theta$.

This we have studied in earlier and we know for such a charge distribution configuration the potential happens to be

$$V(r,\theta) = \frac{P}{3\epsilon_0} r\cos\theta \quad \text{for } r \leq R.$$

$$= \frac{P}{3\epsilon_0}\frac{R^3}{r^2}\cos\theta \quad \text{for } r \geq R$$

We know for spherical coordinates $z = r\cos\theta$

∴ The field inside the sphere i.e, for $r \leq R$ is uniform.

$$\vec{E} = -\vec{\nabla} V = -\frac{P}{3\epsilon_0}\hat{z} = -\frac{1}{3\epsilon_0}\vec{P} \quad \text{for } r \leq R.$$

Outside the sphere i.e, for $r > R$ the potential is

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{P.\frac{4}{3}\pi R^3 \cos\theta}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \quad \text{where, } p = P.\tfrac{4}{3}\pi R^3,$$

the total dipole moment of the sphere

$$= \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{r}}{r^2}$$

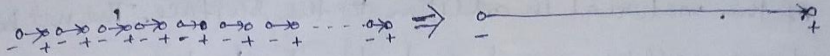
So, the potential outside the sphere is identical to the potential created by a dipole at the centre of the sphere having the same dipole moment as the sphere.

Bound charge

In the calculation of the electric field due to a polarized object we found that the field is the same as generated by charge distribution having surface charge density σ_b and volume charge density ρ_b. 'b' is for bound charge. Where,

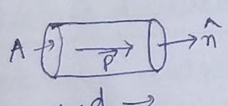
$$\sigma_b = \vec{P}.\hat{n} \quad \& \quad \rho_b = -\vec{\nabla}.\vec{P}$$ where $\vec{P}$ is polarization.

Bound charge densities σ_b and ρ_b are not just mathematical manipulation but they are physical quantities as they represent genuine accumulations of charge. To understand this let us consider a long chain of dipoles as shown in figure. Each dipole has negative charge as tail and



positive charge as head with the dipole moment from negative charge to positive charge as shown by arrow in above figure.

Now, along the length of the chain head of one dipole effectively cancels the tail of its next dipole and vice ~~vesta~~ versa. And finally at the two ends of the chain two charges are left over : a negative charge on the left and a positive charge at the right. As if one electron is removed from right side and one electron is added in the left side of the chain. But in reality no electron can be removed or added neither they can travel through out the length of the chain as all electrons are attached to their specific atom or molecule in a dielectric. ~~the~~ Since the electrons are bound to particular atom and can not be removed we call this charge accumulated at the ends as bound charge. This is just like the normal charge, the word 'bound' is used only for the fact that they can not be removed.



Let us consider a cylinder of crosssection area 'A' and length 'd' of a dielectric material having polarization $\vec{P}$ as shown in the adjascent figure.

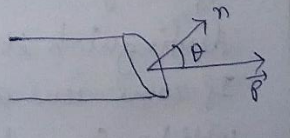
As shown in the case of the chain above $-q$ charge will be accumulated in the left side and $+q$ charge will be accumulated in the right side of the cylinderbecause of polarization. So, the dipole moment of this cylinder is

$$|\vec{p}| = qd \qquad\qquad |\vec{p}| = p$$

$$\text{Now } |\vec{p}| = |\vec{P}|Ad \qquad\qquad |\vec{P}| = P$$

$$\therefore \quad q\cancel{d} = PA\cancel{d}$$

$$q = PA$$



The surface charge density $\sigma_b = \frac{q}{A} = P$

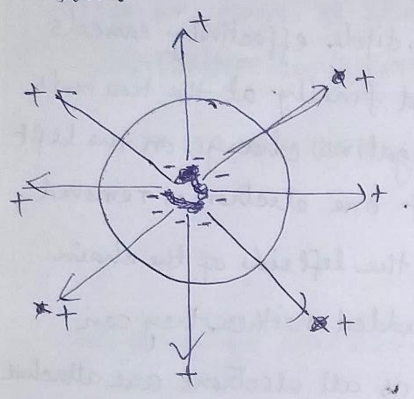
If the area vector (normal to the surface $\hat{n}$) is making angle θ with polarization vector, i.e for oblique surface the effective area is

$$A' = A\cos\theta$$

⑧ $\therefore \quad \sigma_b = \frac{q}{A} = P\cos\theta = \vec{P}\cdot\hat{n}$

So, the polarization will create a layer of bound charge on the surface of the material with bound charge density $\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$.

If the polarization is non-uniform we get accumulation of bound charge in the volume of the material in addition to the surface. The net bound charge in the volume will be equal and opposite to the amount of bound charge pushed out to the surface. The figure below explains this.



Now the surface charge density due to this bound charge accumulation is $\vec{P}\cdot\hat{n}$.

$$\therefore \int_V \rho_b \, d\tau = -\oint_S \vec{P}\cdot d\vec{A}$$

Volume bound charge ↑ Surface bound charge ↑

$$\text{or, } \int_V \rho_b \, d\tau = -\int_V \vec{\nabla}\cdot\vec{P}\, d\tau$$

Applying divergence theorem.

This is true for any volume (arbitrary)

$$\therefore \boxed{\rho_b = -\vec{\nabla}\cdot\vec{P}}$$

Also see example 4.3 in Griffiths.

Gauss's law in presence of dielectric medium

We know that the effect of polarization is to produce accumulations of bound charge within the dielectric $\rho_b = -\vec{\nabla}\cdot\vec{P}$ and on the surface $\sigma_b = \vec{P}\cdot\hat{n}$. If there is other charge in a dielectric apart from these bound charges due to polarization then the total charge density within the dielectric can be written as $\rho = \rho_b + \rho_f$

ρ_f is free charge density. We call it free charge density to distinguish it from the bound charge density due to polarization. If some ions are embedded in the dielectric material then this charge will be free charge.

We know the Gauss's law $\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

Now, $\rho = \rho_b + \rho_f$

$$\therefore \quad \vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}(\rho_b + \rho_f)$$

$$\Rightarrow \quad \epsilon_0 \vec{\nabla}\cdot\vec{E} = -\vec{\nabla}\cdot\vec{P} + \rho_f \qquad \rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\Rightarrow \quad \vec{\nabla}\cdot(\epsilon_0\vec{E}) + \vec{\nabla}\cdot\vec{P} = \rho_f$$

$$\Rightarrow \quad \vec{\nabla}\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_f$$

$$\Rightarrow \quad \boxed{\vec{\nabla}\cdot\vec{D} = \rho_f} \quad -① \quad \text{— Gauss's law in dielectric medium}$$

Where, $\vec{D} = (\epsilon_0\vec{E} + \vec{P})$ is known as electric displacement vector.

Integrating above eq^n over entire volume we get

$$\int_V \vec{\nabla}\cdot\vec{D}\, d\tau = \int \rho_f\, d\tau$$

$$\Rightarrow \quad \boxed{\oint_S \vec{D}\cdot d\vec{A} = Q_f} \quad -②$$

Applying divergence theorem $\oint_S \vec{D} d\vec{A} = \int_V \vec{\nabla}\cdot\vec{D}\, d\tau$.

$Q_f = \int \rho_f\, d\tau$ = total free charge

This is the integral form of Gauss's law in dielectric medium.

Q_f is total free charge enclosed in the volume.

If we know the free charge density or total free charge we can calculate $\vec{D}$ for any dielectric material from above law ① or ②.

~~Displacement~~

Electric displacement vector, $\vec{D}$:

For any dielectric medium the Gauss's law is just like to replace $\epsilon_0\vec{E}$ by $\vec{D}$ and total charge density ρ by free charge density ρ_f. So, one may think that $\vec{D}$ is just like $\vec{E}$ with only difference $\vec{E}$ is originated due to ρ and the source of $\vec{D}$ is ρ_f. But $\vec{D}$ is not at all similar to $\vec{E}$ as there is

① no Coulomb's law for $\vec{D}$.

② curl of $\vec{E}$ is zero, but curl of $\vec{D}$ is not always zero.

$$\vec{\nabla}\times\vec{D} = \epsilon_0\, \vec{\nabla}\times\vec{E} + \vec{\nabla}\times\vec{P} = \vec{\nabla}\times\vec{P}$$

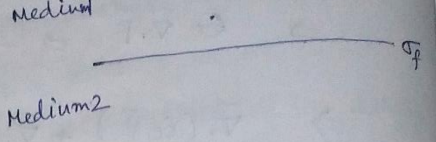
curl of $\vec{P}$ does not vanish in all cases

③ Since curl of $\vec{D}$ is not zero we cannot express $\vec{D}$ as gradient of any scalar. Hence there is no potential for $\vec{D}$.

If there is symmetry $\vec{D}$ can be derived from Gauss's law directly and in such cases $\vec{\nabla}\times\vec{P}$ becomes zero.

Boundary conditions

The electrostatic boundary conditions are



$$\boxed{D_\perp^1 - D_\perp^2 = \sigma_f}$$

If there is no free surface charge

$$D_\perp^1 = D_\perp^2 \quad (\text{when } \sigma_f = 0)$$

So, normal component of displacement vector is continuous when there is no surface free charge.

And $$\boxed{\vec{D}_{\parallel}^1 - \vec{D}_{\parallel}^2 = \vec{P}_{\parallel}^1 - \vec{P}_{\parallel}^2}$$

If there is no polarization in both medium then the parallel component of displacement vector is continuous. This hold also in case of the difference of parallel components of polarization vector becomes zero.

Boundary conditions on $\vec{E}$ are

$$E_\perp^1 - E_\perp^2 = \frac{\sigma}{\epsilon_0}$$

and $$E_{\parallel}^1 - E_{\parallel}^2 = 0$$

Linear dielectrics: permittivity, susceptibility, dielectric constant

The polarization of dielectric material results from an electric field. So, the polarization vector must be a function of electric field. The relation between $\vec{P}$ and $\vec{E}$ follows typical Taylor expansion of $\vec{P}$ in powers of $\vec{E}$.

$$\vec{P} \approx f(E) + f(E^3) + \cdots$$
$$\sim a_0 E + a_1 E^3 + \cdots \quad — ①$$

Magnitude of the co-eff. of expansion reduces drastically for higher order ~~thor~~ terms. In some cases it is negligibly small.

For materials having significant higher order terms of E in polarization are called nonlinear materials. These materials are particularly important for modern day optics. But for materials having negligibly small higher order components expect from the first term in above eq^n ① are called linear dielectrics where polarization is linearly proportional to electric field. When the electric field is not too strong they canbe related as

$$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}} \qquad \text{(for linear dielectric)}$$

ϵ_0 is absolute permittivity of free space.

χ_e is called the electric susceptibility of the medium. The value of χ_e is dependent on the microscopic structure of the material.

Here the electric field $\vec{E}$ is the total electric field i.e, due to free charge as well as polarization it self.

For linear dielectric medium displacement vector

$$\boxed{\vec{D} = \epsilon_0 \vec{E} + \underline{\vec{P}} = \epsilon_0 \vec{E} + \underline{\epsilon_0 \chi_e \vec{E}} = \epsilon_0 (1+\chi_e)\vec{E}}$$

Now, $\vec{D}$ is also proportional to $\vec{E}$ as we can see from above eqn.

$$\therefore \boxed{\vec{D} = \epsilon \vec{E}}$$

where, $\boxed{\epsilon = \epsilon_0 (1+\chi_e)}$

ϵ is called the permittivity of the material.

In vacuum there is no medium so there is no polarization. Hence χ_e is zero and the permittivity is ϵ_0. This is why ϵ_0 is called permittivity of free space.

Susceptibility χ_e is dimensionless quantity.

Permittivity has unit $C^2/N\text{-}m^2$.

$$\boxed{\frac{\epsilon}{\epsilon_0} = 1 + \chi_e = \epsilon_r}$$

ϵ_r is called the relative permittivity or dielectric constant of the material.

See example 4.5 in Griffiths.

Homogeneouls dielectric medium is for which susceptibility does not vary with position.

For homogeneous linear dielectric medium ~~the~~ ϵ_0 should be replace by ϵ to get the field $\vec{E}$.

Like the field due to point charge q (free charge) embedded in a homogeneous linear dielectric medium is

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}}$$

Similarly in the expression of force ϵ_0 is to be replaced by ϵ.

For crystals in general, polarization is not the same in all directions.
In such cases the polarization is given by

$$P_x = \epsilon_0 \left(\chi_{exx} E_x + \chi_{exy} E_y + \chi_{exz} E_z \right)$$

$$P_y = \epsilon_0 \left(\chi_{eyx} E_x + \chi_{eyy} E_y + \chi_{eyz} E_z \right)$$

$$P_z = \epsilon_0 \left(\chi_{ezx} E_x + \chi_{ezy} E_y + \chi_{ezz} E_z \right)$$

In this case susceptibility has nine coefficients that form a tensor i.e. in this case susceptibility is a tensor of rank 2.
Such medium where the properties are different in different directions are called anisotropic medium.
Isotropic medium is the one for which the properties are same in all directions.