VIVEKANANDA COLLEGE
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NAAC ACCREDITED ‘A’ GRADE

Topic: Largest Eigen-value Problem

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Largest Eigen-value and Corresponding Eigen-vector of a matrix

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C     Largest Eigen value and corresponding Eigen vector of a matrix by power method using Subroutine
      Dimension a(20,20), v(20)
      Write(*,*)"Enter the order of matrix and accuracy"
      Read(*,*)n, e
      Write (*,*)"Enter the matrix"
      Do i = 1,n
      Read(*,*)(a(i,j), j = 1,n)
      End do
      Write(*,*)"Enter the initial vector"
      Read(*,*)(v(i), i = 1,n)                          ! Consider initial eigen-vector [1 0 0] always

      Call Cal_Vector (n,a,v)
      Call Cal_Eigen (n, v, eigenold)
      it = 1
      Write(*,1)"Iteration no.: ", it, "  Eigen value: ", eigenold,     ! 1 in place of second * is for formatting
     &  "  Vector: [",(v(i), i=1,n),"]"
      Do                                                !Infinite do loop
      Call Cal_Vector (n,a,v)                           !Carefully note that first you have to call "Cal_vector"
      Call Cal_Eigen (n, v, eigennew)                   ! Secondly you call "Cal_Eigen"
      it = it +1
      Write(*,1)"Iteration no.: ", it, "  Eigen value: ", eigennew,
     &  "  Vector: [",(v(i), i=1,n),"]"
      If (abs(eigennew-eigenold).lt.e)then              ! You reach at your result
       Exit                                             ! As You reach at your result, so exit
       End if
       eigenold = eigennew  !If (abs(eigennew-eigenold).gt.e)then store eigennew at eigenold and again calculate eigennew
       end do                                           ! End of infinite do loop
       Write (*,1)"No. of iterations: ", it, "  Largest eigen value: ",
     &   eigennew, "  Eigen vector: [",(v(i), i=1,n),"]"
1      Format (A, I3, A, F8.4, A, 3F8.4, A)             ! A is used to format all values in string
       Stop
       End

       Subroutine Cal_vector(n,a,v)                     ! Initial vector "v", matrix "a" is called here from main program
       Dimension a(20,20), v(20), c(20)
       Do i = 1,n
        c(i)=0.0
        End do
        Do i = 1,n
        Do j = 1,n
       C(i)= c(i)+ a(i,j)*v(j)                          !Multiply the initial vector "v" with the matrix "a" and get new vector "c"
       End do
       end do
       do i = 1,n
       v(i) = c(i)                                      ! Store new vector "c" at "v"
       end do
       Return                                           !Return new vector "v" to the main program by calling subroutine "Cal_vector"
       End

       Subroutine Cal_eigen(n, v, eigen)  !new vector "v" which already returned to main program from "Cal_vector" is called
here
       Dimension v(20)
       eigen = v(1)                                     ! consider new variable "eigen" is the first component of "v"
       do i = 1,n
       If (eigen.lt.v(i))then
        eigen = v(i)                                    ! Maximum component of the vector "v" being equal to eigen value
        End if
        End do
        do i = 1,n
        v(i)= v(i)/eigen               ! Divide each vector-component with max. vector-component to normalize the eigen vector
        end do
        Return                          !Return the final vector "v" and eigen value (eigenold, eigennew) to the main program
        end
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The above mentioned program is to obtain largest eigen value of a matrix and its corresponding eigen vector by using **Power method**.

Let our matrix be A = $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}$

Before today, we evaluate eigen values of a matrix *A* by using characteristic equation: $(|A| - \lambda|I|)\,\boldsymbol{X} = 0$, where $|A|$ is the determinant of matrix *A* and *I* is the unit matrix and λ is supposed to be the eigen value and $\boldsymbol{X}$ is the corresponding eigen vector of eigen value λ.

In the power method, we start by considering an initial eigen vector $\boldsymbol{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and calculate $A*\boldsymbol{X}$. Lets see what happen…….

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} * \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a11 \\ a21 \\ a31 \end{pmatrix} = \boldsymbol{C}$$

Then we have to normalize $\boldsymbol{C}$ vector. To normalize a series of numbers we have to divide all the numbers by the maximum number of that series. So in present case, we have to see the maximum element of the vector $\boldsymbol{C}$ and then divide every component of $\boldsymbol{C}$ by the maximum component. Suppose, in present case, $a21$ is max. among $a11$, $a21$ and $a31$ (three component of $\boldsymbol{C}$). Then, $\boldsymbol{C}$ vector becomes,

$\boldsymbol{C} = a21 \begin{pmatrix} a11/a21 \\ 1 \\ a31/a21 \end{pmatrix} = a21 * \boldsymbol{E}$, where, $a21$ is the eigen value and $\boldsymbol{E}$ is the eigen vector of first iteration. Then for 2nd iteration, we have to do, $A*\boldsymbol{E}$ and again we will get, eigen value and eigen vector of 2nd iteration and the loop will be continued until the eigen value becomes converged. Then finally we will obtain the largest eigen value and corresponding eigen vector.

Try the above program with

1. $\begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$ Ans: Largest Eigen value: 5.9998, Eigen vector:[0.0625 0.3750 1.0000]

2. $\begin{pmatrix} 9 & 10 & 8 \\ 10 & 5 & -1 \\ 8 & -1 & 3 \end{pmatrix}$ Ans: Largest Eigen value: 19.2860, Eigen vector:[1.0000 0.6685 0.4502]

3. $\begin{pmatrix} 25 & 1 & 0 \\ 2 & 1 & 3 \\ 3 & 6 & 2 \end{pmatrix}$ Ans: Largest Eigen value: 25.1024, Eigen vector:[1.0000 0.1024 0.1564]

Accuracy = 0.0005, Initial vector = [1 0 0]

Note that output values are written using format