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MINKOWSKI SPACE

I. RELATIVITY PRINCIPLE AND SPACE-TIME GEOMETRY

In this note we will discuss a few aspects of the Minkowski space which represents the exact geometry of space and time in absence of gravity. This means Minkowski space represents the geometry of space and time when a particle is subjected to all the other three interactions, namely the strong, weak and electromagnetic interactions irrespective of the magnitude of the velocity of the particle. Minkowski space is different from our common sence notion about the geometry of space and time which is built upon low-energy mechanical phenomenon. We will later find that the geometry of space and time is described by Riemannian geometry (more appropriately pseudo-Riemannian geometry) when we include gravity.

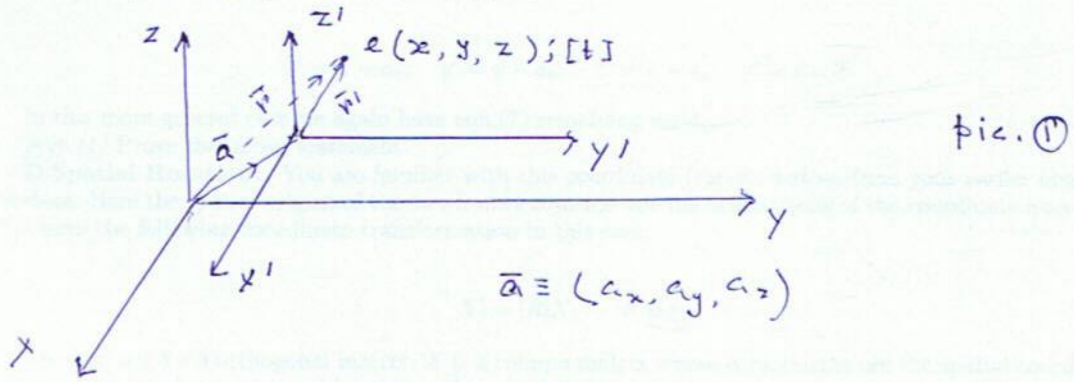
You know from your elementary relativity course that the low-energy mechanical phenomena are described by the Newtonian mechanics where the tarjectory of a particle, subjected to a given force, is determined by the three well-known laws of Newton. However to decribe the motion of a particle we have to introduce a coordinate system together with a set of synchronized clocks. The clocks are synchronized in such a way that different clocks at different positions assign the same time to a given event. You can look at references [1,2] for a detail description of clock synchronization process in both Galilean relativity and Special relativity.

If we choose our coordinate system arbitrarilly, we will find that even the simplest motions, like the motion of a free particle, appearing to be complicated. This is due to the inertial forces. It will also appear that the magnitude and direction of the inertial forces change with space as well as time. Thus space and time will appear inhomogeneous and and anisotropic. To understand this, consider the centrifugal force and the coriolis force present with respect to an earth-bound observer, i.e, an observer whose coordinate system is fixed with the Earth. The magnitude of the centrifugal force changes with the distance from the centre of the earth and thus the space no longer appears to be homogeneous to the observer.

On the other hand, experience shows that there exists a class of observers with a corresponding set of coordinate systems and synchronized clocks with respect to which mechanical phenomena take the simplest form. There are no inertial forces present in these refernce frames. Space and time appear to be homogeneous and isotropic with respect to such observers. These refernce frames are known as the inertial frames. Experimentally, a refernce frame at rest with respect to an observer who is far away from all the sources of forces give a good example of an inertial frame. However the problem is that there exist not a single inertial frame but a multitude of them forming a class. When you consider the motion of a given particle with respect to different inertial frames, you will find that the motion is goverened by physical laws of the same form although the magnitude (and the direction also depending on the physical quantity and the inertial frames) of some of the physical quantities describing the motion will change. The purpose of a relativity principle is to predict and explain which physical quantities will change and which physical quantities will remain the same when you compare the motion of the same system with respect to two different inertial frames. This is achieved by a set of relations known as the coordinate transformation laws. These give the relation between the space-time coordinates of a given event assigned by two diiefernt inertial frames. In effect, these coordinate transformation laws give us the underlying geometry of space and time and govern the nature of the physical laws through the **principle of covariance**. The physical/geometrical quantities which do not change when we go from one inertial frame to another is known as the **invariants** of the relativity principle. These afe also known as the **scalars**.

In three dimensions each inertial frame is associated with a three dimensional Cartesian coordinate system and a set of synchronized clocks. Experiments show that two inertial frames may differ from each other in the following ways:

A.Spatial Translation The origins of the spatial coordinate systems may be displaced with respect to each other by an arbitrary vector as shown below.



In this case we have the following relations between the coordinates of the event e given in the picture:

$$\vec{r'} = \vec{r} - \vec{a}; \quad (1)$$

and

$$t' = t \quad (2)$$

In other words:

$$x' = x - a_x; \quad y' = y - a_y; \quad z' = z - a_z; \quad t' = t. \quad (3)$$

Here we are assuming that the clocks of the two observers are adjusted in such a way that they agree with each other for any event. Since the two observers are at rest with respect to each other, this is expected from the homogeneity and isotropy of space with respect to an inertial observer. Note that we have the follwing important relation between the spatial and temporal interval between any two events as measured by the two observers:

$$(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad (4)$$

Together with

$$t_2' - t_1' = t_2 - t_1 \quad (5)$$

This tells us that the spatial interval between two events is an invariant under the coordinate transformations given by eqn.(3) provided the spatial interval is measured using the Eucledian geometry for space. The temporal interval, defined by eqn.(5) is remainig invariant separately.

B.Temporal Translation This is a tranformation which involve translation of the time coordinate only and is given by:

$$x' = x; \quad y' = y; \quad z' = z; \quad t' = t + T. \quad (6)$$

This is achieved if we set $t' = T$ when $t = 0$ for any event. We can consider a pair of events and we have again,

$$\Delta r'^2 = \Delta r^2; \quad \Delta t' = \Delta t' \quad (7)$$

Where Δr^2 and Δt are given by eqns.(4) and (5).

C.Space-Time Translation: Lastly, we can compose the spatial and temporal translation to have the following more general transformation,

$$x' = x - a_x; \quad y' = y - a_y; \quad z' = z - a_z; \quad t' = t - T. \tag{8}$$

In this more general case we again have eqn.(7) remaining valid.

prob.(1) Prove the above statement.

D.Spatial Rotation: You are familiar with this coordinate transformation from your earlier course on vectors. Here the spatial origins of the two frames coincide but the orientations of the coordinate axes differ. We have the following coordinate transformation in this case:

$$X' = [R]X; \qquad t' = t \tag{9}$$

Here $[R]$ is a 3×3 orthogonal matrix, X is a column matrix whose components are the spatial coordinates of the corresponding event and is expressed in the following way:

$$X \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \qquad \tilde{X} = X^T \equiv (x \; y \; z).$$

prob.(2) Using the orthogonality of $[R]$, show that eqn.(7) is again valid for the coordinate transformation given by eqn.(9).

prob.(3) Write down the transformation equations explicitly when the two frames are rotated about the common Y- axis by an angle ϕ. Repeat prob.(2) for this $[R]$.

prob.(4) The most general coordinate transformation between two frames without having any relative motion is a combination of the transformations (C) and (D). Show that eqn.(7) is valid for this general case.

E. Galilean Transformation: This is the most important transformation that we have to discuss. Here the S'-frame is moving with respect to the S-frame with a uniform velocity along a certain direction. We can take the direction of motion along the common $X - X'$ axes. We also assume that at $t = t' = 0$ the origins were coincident. We tell that the two frames are in **standard configuration**. The coordinate transformation rule is given by the following expression:

$$x' = x - vt; \quad y' = y; \quad z' = z; \quad t' = t. \tag{10}$$

Here the motion of S' is along the positive X-direction with magnitude v. We note that, provided the two frames are in standard configuration, there is no change in the time coordinate. This is what we mean by "absolute time". You can look at [1,2] for further discussions. For our purpose, it is important to note the following:

(1) The temporal interval between a pair of events remain the same in both S and S'. This is an outcome of the absoluteness of time.

(2) The spatial interval between two simultaneous events remain the same in both S and S'. Thus we have again eqn.(7) remaining valid for a pair of simultaneous events. This is because of the absoluteness of simultaneity which is a consequence of the absoluteness of time.

Thus the spatial geometry remained to be Euclidean as were observed earlier. For non-simultaneous pair of events the spatial interval depends on the temporal interval. The temporal interval again remain absolute and the same in all the inertial frames.

Before the advent of special relativity, it was believed that the laws of motions of a particle will remain the same in all the inertial frames and are given by the three laws of Newton. The transformations of the different physical quantities were obtained using eqn.(10). To illustrate, eqn.(10) tells that the acceleration of a particle remain the same in both S and S'. Thus in the right hand side of Newton's second law we should use an expression for the force which is invariant under the transformation eqn.(10). This is the case with the Newtonian gravity and the electrostatic force which are obtained from the low-velocity experiments. The invariance of the laws of motion under the transformation law eqn.(10) is known as the Galilean Relativity priciple. It was then obvious to extend the principles of Galilean relativity to the complete set of electromagnetic interactions. However, electrodynamics deals with the propagation of electromagnetic radiations which travell with the velocity of light. This is high compared to our day-to-day mechanical experiments and as you know discrepancies with the Galilean relativity were observed in many optical experiments.

Einstein solved the problem by changing the transformation law given by eqn.(10). He retained the principle of relativity and extended it to all the laws of physics through his first postulate. He changed the coordinate transformtion laws from the Galilean to the Lorentz transformation laws. In the next section we will discuss the Lorentz transformation laws and find that the underlying space-time geometry is given by the Minkowski space.

II. SPECIAL RELATIVITY AND MINKOWSKI SPACE

We again consider two inertial reference frames S and S' in standard configuration with the relative velocity along the common $X - X'$ axes. We know that the transformation laws that leads us to a relativity principle for the optical phenomena are given by the Lorentz transformation laws. We have,

$$ct' = \frac{ct - \beta x}{\sqrt{1-\beta^2}}; \quad x' = \frac{x - \beta ct}{\sqrt{1-\beta^2}}; \quad y' = y; \quad z' = z \tag{11}$$

It can be shown easilly that neither the spatial interval between two simultaneous events in S nor the temporal interval between two events remain invariant when go from S to S'. Here the spatial interval is evaluated using the Euclidean expression with the Cartesian coordinates. It is also obvious from the transformation laws that time is no longer absolute. These are discussed in detail in [1,2].

However it is straightforward to show that we have the following relation remaining valid between the coordinate intervals for an arbitrary pair of events:

$$c^2(t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 \tag{12}$$

and,

$$(y'_2 - y'_1) = (y_2 - y_1); \quad (z'_2 - z'_1) = (z_2 - z_1) \tag{13}$$

We can combine the above two equations to have,

$$c^2(t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 - (y'_2 - y'_1)^2 - (z'_2 - z'_1)^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 \tag{14}$$

This relation holds for an arbitrary Lorentz transformation in an arbitrary direction. In the above expression we have chosen the temporal interval to come with the posituve sign and the spatial intervals to come with negative signs. We can choose the opposite, that is, we can also show that,

$$-c^2(t'_2 - t'_1)^2 + (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 = -c^2(t_2 - t_1)^2 + (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \tag{15}$$

The following discussions do not change apart from a few changes of signs. In this notes we will follow the first convention.

We note that eqn.(14) is similar to eqn.(4) apart from two changes:

(1) We have added a new dimension to the spatial dimensions given by the coordinate ct. Thus in place of discussing three dimension spatial geometry, we now consider four dimensional space-time geometry.

(2) The geometry of the four dimensional space-time is no longer Euclidean. If the four dimensional space-time geometry would have been Euclidean we would have the following expression for the square of the distance between a pair of events:

$$(\Delta r)^2 = c^2(t'_2 - t'_1)^2 + (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2$$

Where all the quadratic terms come with a positive sign.
We now define the **interval**, Δs, between two events by the following expression:

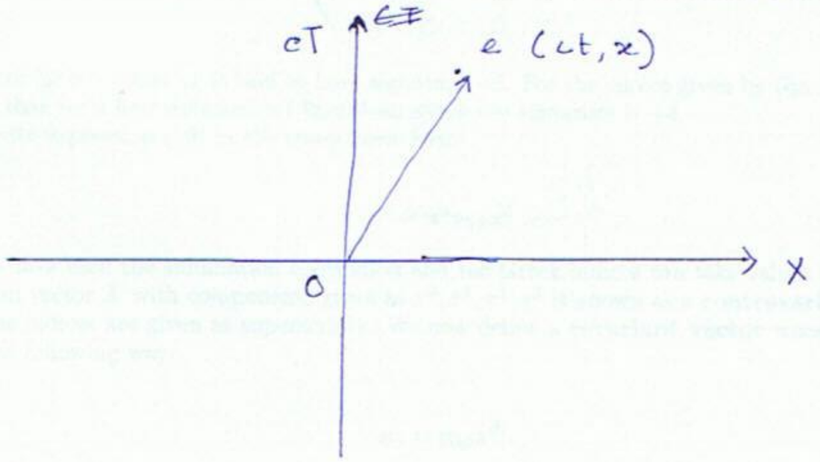
$$(\Delta s)^2 = c^2(t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 - (y'_2 - y'_1)^2 - (z'_2 - z'_1)^2 \tag{16}$$

This is analogous to the square of the distance between two points in the ordinary Euclidean spaces. However the square of the interval can be positive, zero or negative since some of the squares in the above expression come with negative signs.

The geometrical spaces where the square of the interval or the square of the 'distance' between two points are of the similar form as the Euclidean spaces (we have only square of the coordinate intervals in the expression for the square of the interval between two points) but can be positive, zero or negative are known as the pseudo-Euclidean spaces.

Thus the Minkowski space is a four-dimensional pseudo-Euclidean space with three spatial dimension and one temporal dimension. The inertial frames are used to coordinatize the different events in this space. The interval between two events given by either eqn.(14) or (15). The Lorentz transformations relate the space-time coordinates of a given event as assigned by different inertial frames. The Lorentz transformations act on the coordinates in such a way that the interval between any pair of events remain the same under the Lorentz transformations. This is similar to rotations in three dimensional Euclidean space.

We represent Minkowski space pictorially in the following way which is known as the Minkowski diagram:



Here the origin represent an event which has occured at $t = 0$ at the spatial point $(0, 0, 0)$. The Y- and the Z- axes are passing through the origin. A point in the page,e, represents an event whose X-coordinate is x and time coordinate is ct. Note that the later has the dimension of length. Some author only use t in place of ct. We have shown how to find the coordinates.

We should note two aspects:

(1) Minkowski space is a four dimensional space while in the diagram we only represent the ct and the x axes suppressing the other two dimensions.

(2) The geometry of the paper is a two dimensional Euclidean geometry while the actual $cT - X$ geometry is a two dimensional pseudo-Euclidean geometry. This leads to a few interesting paradoxes that we need not to bother about in this elementary course.

III. FOUR VECTORS

We will now discuss a few geometrical aspects of the Minkowski space.

As in the three dimensional Euclidean space, we can consider a radius vector joining the origin and the event e. It has coordinates (ct,x,y,z) and is known as a **Four vector**. The corresponding interval is given by the following expression:

$$s^2 = (ct)^2 - x^2 - y^2 - z^2 \tag{17}$$

We now change our notation. We replace the coordinates (ct, x, y, z) by (x^0, x^1, x^2, x^3). We then have,

$$s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \tag{18}$$

We write it in the following way:

$$s^2 = \tilde{X}[g]X \tag{19}$$

Here X is a 4×1 column matrix with the componets being x^0, x^1, x^2, x^3. $\tilde{X}$ is the corresponding transposed row matrix: $(x^0x^1x^2x^3)$. $[g]$ is a 4×4 matrix, known as the metric tensor and is given by the following expression:

$$[g] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The trace of $[g]$ is -2 and $[g]$ is said to have signature -2. For the choice given by eqn.(15), the signature is $+2$. Note that for a four dimensional Euclidean space the signature is $+4$.
We can write expression (19) in the component form:

$$s^2 = x^\alpha g_{\alpha\beta} x^\beta \tag{20}$$

Where we have used the summation convention and the Greek indices can take values $0, 1, 2, 3$.
The column vector X with components given as x^0, x^1, x^2, x^3 is known as a **contravariant four vector**. Note that the indices are given as superscripts. We now define a **covariant vector** whose components are defined in the following way:

$$x_\alpha = g_{\alpha\beta} x^\beta \tag{21}$$

Thus we have:

$$x_0 = x^0; \quad x_1 = -x^1; \quad x_2 = -x^2; \quad x_3 = -x^3 \tag{22}$$

Note that you can express a contravariant vector both as a column or row vector. Similar point is valid for the covariant vectors. However you should be careful about the superscripts, subscripts and the signs. Covariant vector are expressed with their components labelled by subscripts while contravariant vectors have their components labelled by superscripts. We can now express eqn.(19) or eqn.(20) as:

$$s^2 = x_\alpha x^\alpha \tag{23}$$

So far we have discussed only the interval associated with a single four vector. This is similar to the square of the magnitude of Euclidean vector. We now generalize the concept of the scalar product to the four vectors. Let us consider two contravariant four vectors **A** and **B** with components given by: a^0, a^1, a^2, a^3 and b^0, b^1, b^2, b^3 respectively. The inner product between these two four vectors is given by,

$$\mathbf{A}.\mathbf{B} = a^\alpha g_{\alpha\beta} b^\beta = a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3 = a_\alpha b^\alpha \tag{24}$$

When we obtain the covariant vector from a contravariant vector, we say that we are lowering the indices of the cotravariant vector using the metric which is a 2nd rank covariant tensor. This is why the indices are given as subscripts. We can now define the contravariant metric tensor with components given by $g^{\alpha\beta}$ such that the following relation is valid:

$$\mathbf{A.B} = a_\alpha g^{\alpha\beta} b_\beta = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3 = a_\alpha b^\alpha \tag{25}$$

Where again **A** and **B** are now two covariant four vectors. You can show from the above expression that the contravariant metric tensor is gien by the same matrix as the covariant metric tensor. However this is only valid for Euclidean or pseudo-Euclidean spaces with Cartesian coordinates used to express the distance or the interval respectively. In general we have the following relation valid:

$$g_{\alpha\beta} g^{\beta\gamma} = \delta_\alpha^\gamma \tag{26}$$

Where δ_α^γ is the Kronecker delta which is the same as the identity matrix.

Prob.5 Find out the expressions for the covariant and contravariant metric tensors in three dimensional Euclidean space using the spherical polar coordinates.

IV. GEOMETRY OF THE MINKOWSKI SPACE

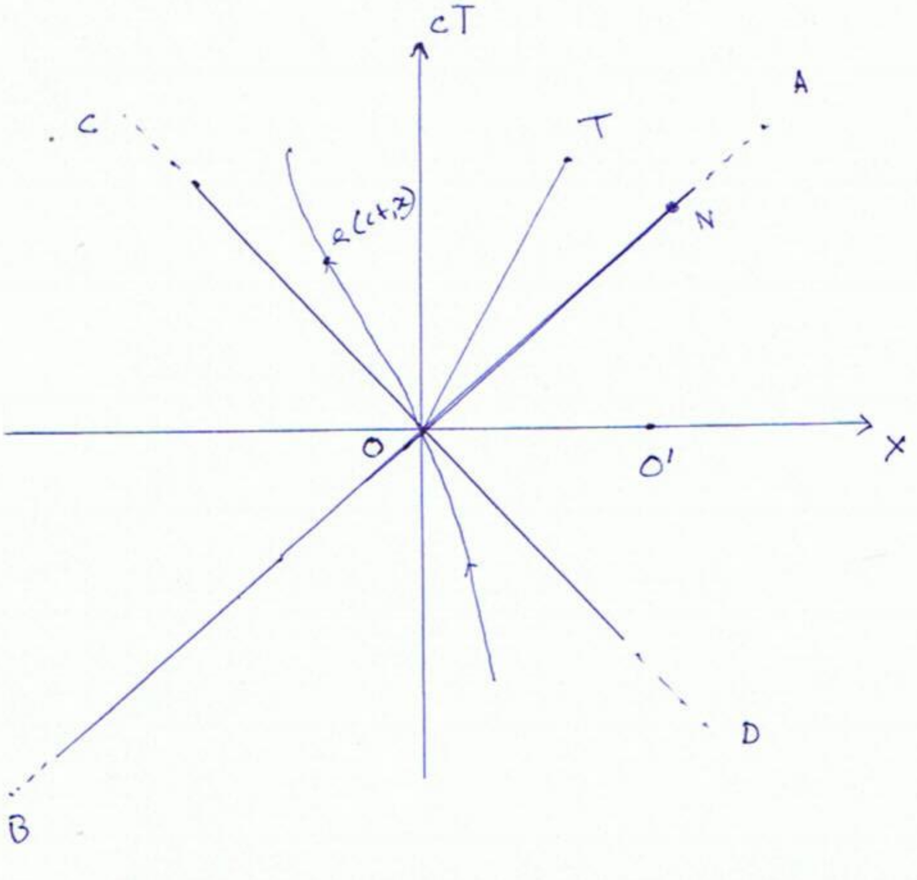
In this section we will discuss the non-trivial consequences of the pseudo-Euclidean character of the Minkowski space. We will define timelike, sapcelike and null vectors in the Minkowski space. We will also discuss the world lines of point particles and proper time.

Minkowski diagram:

Let us consider the interval corresponding to a four vector with components (x^0, x^1, x^2, x^3). Note that to represent the components we use the comma while to represent a row matrix we do not use the comma. The square of the interval is given by,

$$s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \tag{27}$$

Depending on the relative values of the components we can have three classes of four vectors known as the null or lightlike vectors, timelike vectors and spacelike vectors. We will use the following diagram to discuss these classes. This diagram is known as the **Minkowski diagram**.



In this diagram we plot $x^0 = ct$ along the Y- axis and $x = x^1$ along the X- axis. The origin O represents the event $(0,0,0,0)$ although we have suppressed the Y and Z axes in this two dimensional diagram. We can also regard this event as $(0,0)$. A curve in this Minkowski diagram represents an equation of the form:

$$x = f(ct) \quad or \quad x^1 = f(x^0). \tag{28}$$

and thus give us the trajectory of a particle. This is known as the **world line** of a particle. The world line of a free particle moving with uniform velocity is represented by a straight line. We then have,

$$\beta = \frac{x^1}{x^0} = \frac{x}{ct} \tag{29}$$

Thus β is given by the inverse of the slope of the straight world line. You can fix the units by choosing *1cm = one light-second.* This has the advantage of representing the the world line of a light signal or a photon moving with velocity c by a straight line inclined to the X- axis by 45°. In the above diagram the ray OT represent the world line of a particle moving with velocity $v < c$ which has started its motion from the event O, i.e. from $x = 0$ at $t = 0$.

Prob.(6) Discuss the nature of the world lines represented by the rays ON and OS. Can OS represent the motion of any physical particle/signal? The ray ON is making an angle 45° w.r.t the X-axis.

Note that when we consider the motion of a particle subjected to a force field the world line will no longer be a straight line and will be represented by a curve. We can draw a tangent to such a curvilinear world line at an arbitrary event (ct, x). The event indicates the arrival of the particle at x at the time t and is represented by the point e in the diagram. The slope of the tangent at every point on the curved world line should be greater than 45° so that it can represent the motion of a massive particle.

Prob.(7) Draw all possible world lines of a photon that is passing through the event O' as shown in the above diagram.

Null four vectors:

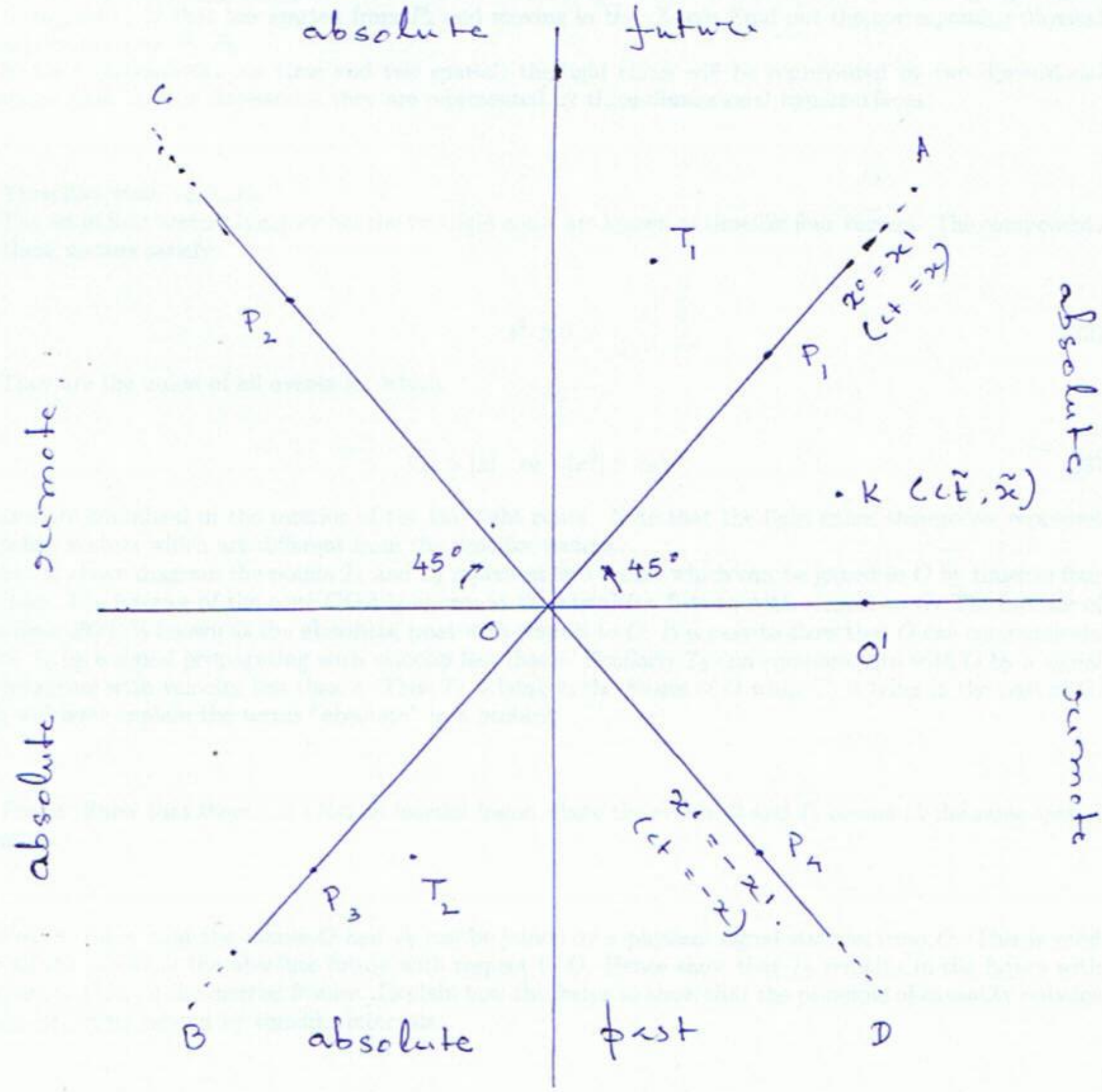
This is the case when

$$s^2 = 0 \tag{30}$$

In the following diagram (Minkowski diagram) the two straight lines

$$ct = \pm x \quad or \quad x^0 = \pm x^1 \tag{31}$$

represent the set of all null vectors.



The set of four vectors joining the origin and the set of events P_1, P_2, P_3 and P_4 are all null four vectors having $s^2 = 0$. You should note that in the Euclidean space the only null vector is $(0,0,0)$ since the norm is given by $x^2 + y^2 + z^2$. The negative signs in eqn.(27) can give $s^2 = 0$ even with finite values of the coordinates.

The set of the null vectors in the Minkowski space are also known as the lightlike vectors. The two straight lines $x = \pm ct$ or $x^1 = \pm x^0$ are said to form the **Light cones**. The portion of the light cone for $t \geq 0$ is known as the **future light cone** while the portion with $t \leq 0$ is known as the **past light cone**. They are also known as the **outgoing** and the **incoming** light cones respectively.

The straight lines $x = \pm ct$ or $x^1 = \pm x^0$ represent the trajectories of light signals (say a photon) that pass through the origin at $t = 0$. The ray AB represent a light signal that is originated at $x = -\infty$ at $t = -\infty$ and is passing through the spatial origin at $t = 0$, i.e, the event O. The ray CD represent a light signal that is originated at $x = \infty$ at $t = -\infty$ and is passing through the spatial origin at $t = 0$, i.e, the event O. In the above diagram the cone AOC represents the future lightcone while the cone BOD represents the past lightcone. The event P_1 is said to be on the outgoing or the future light cone. The event O can communicate with P_1 by an outgoing light signal starting from O and moving in the +X-axis. The event P_4 is said to be on the incoming or past light cone. The event P_4 can communicate with O by an incoming light signal with respect to O that has started from P_4 and moving in the -X axis.Find out the correspondng physical interpretations for P_2, P_3.

In three dimensions (one time and two spatial) the light cones will be represented by two dimensional surfaces while in four dimensions they are represented by three dimensional hypersurfaces.

Timelike four vectors:

The set of four vectors lying within the two light cones are known as timelike four vectors. The components of these vectors satisfy:

$$s^2 > 0 \tag{32}$$

They are the union of all events for which,

$$|ct| > |x| \quad or \quad |x^0| > |x^1| \tag{33}$$

and are contained in the interior of the two light cones. Note that the light cones themselves represent lightlike vectors which are different from the timelike vectors.

In the above diagram the points T_1 and T_2 represent two events which can be joined to O by timelike four vectors. The interior of the cone COA is known as the **absolute future** with respect to O. The interior of the cone BOD is known as the **absolute past** with respect to O. It is easy to show that O can communicate with T_1 by a signal propagating with velocity less that c. Similarly T_2 can communicate with O by a signal propagating with velocity less than c. Thus T_1 is lying in the future of O while T_2 is lying in the past of O. We will later explain the terms "absolute" in a problem.

Prob.8 Show that there can exist an inertial frame where the events O and T_1 occurs at the same spatial point.

Prob.9 Show that the events O and T_1 can be joined by a physical signal starting from O. This is valid for all the events in the absolute future with respect to O. Hence show that T_1 remains in the future with respect to O in all the inertial frames. Explain how this helps to show that the principle of causality remains valid for events related by timelike intervals.

Spacelike four vectors:

The set of four vectors lying outside the two light cones are known as spacelike four vectors. The components of these vectors satisfy:

$$s^2 < 0 \tag{34}$$

They are the union of all events for which,

$$|ct| < |x| \quad or \quad |x^0| < |x^1| \tag{35}$$

and are contained in the exterior of the two light cones. These vectors are contained within the regions AOD and COB in the above diagram. Again the light cones themselves are excluded.

In the above diagram the points K represents an event which can be joined to O by spacelike four vectors. The spacelike events with respect to the event O are joind to O by spacelike four vectors. They represent motions with speed greater than that of light. According to the second postulate of special relativity, O can never communicate with K since physical signals can not propagate with a velocity greater than that of light. Thus all the events in the exterior region of the two light cones are known as absolutely remote with respect to the event O. Note that spacelike events are not further distinguised by 'future' or 'past'. The reason is illustrated in the following problem.

Prob.(10) Consider the spacelike event K in the above diagram. The coordinate are $(c\bar{t}, \bar{x})$ with $\bar{t} > 0$. Thus this event lies in the future of O in the inertial frame with axes cT, X. Show that in this case $\beta > 1$. Hence show that you can find an inertial frame with axes cT', X', such that $\bar{t}' < 0$. Thus with respect to the S' observer, K happens in the past of the event O. Thus, for spacelike events with respect to O, future and past w.r.t O depends on the inertial frame we choose. How you are going to explain causality in this case?

Prob.(11) Consider the event O' as shown in the diagram.
(i) Draw the light cones with respect to O'.
(ii) Show the points that are lightlike w.r.t both O and O'.
(iii) Shade the regions that are absolutely future and absolutely past with respect to both O and O'.
(iv) Can O and O' communicate with each other.
Prob.(12) All the university problems.

V. REFERENCES

[1] Special Relativity: Robert Resnick
[2] Special Relativity: A. P. French