VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED 'A' GRADE

Topic: Force and Torque on Magnetic Dipole (https://www.youtube.com/watch?v=s5UUlAyVrIw)

Course Title: Electricity and Magnetism

Paper: PHS-A-CC-2-3TH

Unit: 2

Semester: 2

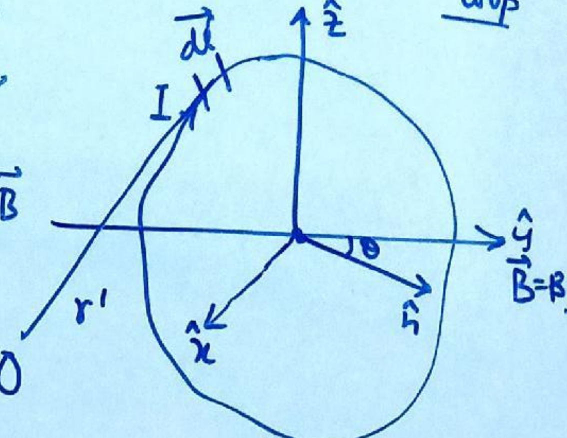
Name of the Teacher: Arvind Pan

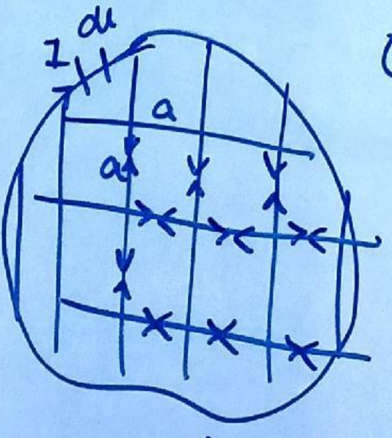
Name of the Department: Physics UG

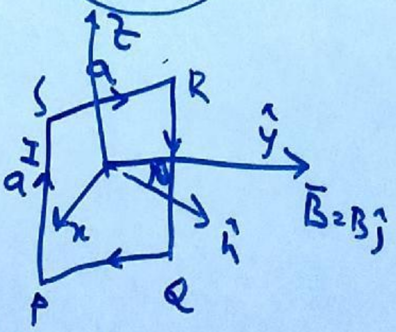
Force & Torque acting on a magnetic dipole / current loop

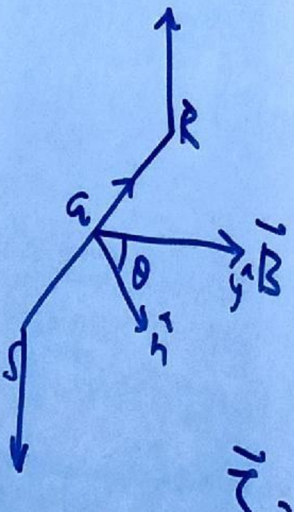
$$d\vec{\tau} = \vec{r}\,' \times \underbrace{I\,\vec{dl} \times \vec{B}}_{d\vec{F}}$$

$$\vec{\tau} = \int \vec{r}\,' \times I\,\vec{dl} \times \vec{B}$$









Force on the element PQ & RS are equal & opposite & act along the same line (z axis)

Force on RS $= Ia(-\hat{i}) \times B\hat{j} = IaB(-\hat{k})$

Force on PQ $= Ia(\hat{i}) \times B\hat{j} = IaB(\hat{k})$

$$\vec{\tau} = a\hat{i} \times IaB(\hat{k})$$

$$= \underline{Ia^2}\,\hat{i} \times B\hat{k}$$

$$= \vec{m} \times \vec{B}$$

$\vec{\tau} = \vec{p} \times \vec{E}$

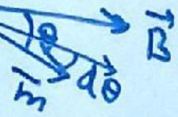
$\vec{p}$ → electric dipole

$\vec{E}$ → Electric field.

Force & Torque acting on a magnetic dipole / current loop

$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}} \Rightarrow mB\sin\theta = |\tau|$$

$$dU = \tau \, d\theta$$



$$= mB\sin\theta \, d\theta$$

$$U = \int_{\theta_0}^{\theta} mB\sin\theta \, d\theta = mB\left|-\cos\theta\right|_{\theta_0}^{\theta}$$

$$= mB\left|\cos\theta_0 - \cos\theta\right|$$

for $\theta_0 = \pi/2$

$$U = -mB\cos\theta = -\vec{m}\cdot\vec{B}$$

Electrostat: $U = \vec{p}\cdot\vec{E}$

Force $\vec{F} = -\nabla U = \nabla(\vec{m}\cdot\vec{B})$

$$= (\vec{B}\cdot\nabla)\vec{m} + (\vec{m}\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{m}) + \vec{m}\times(\nabla\times\vec{B})$$

for $\vec{m}$ = constt & curl free $\vec{B}$ for magnetic dipole $\nabla\times\vec{B} = 0$

$$\boxed{\vec{F} = (\vec{m}\cdot\nabla)\vec{B}}$$

Electrostat: $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$