VIVEKANANDA COLLEGE
THAKURPUKUR
KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: Statistics of Paramagnetism (https://www.youtube.com/watch?v=Rq-A740T6lM
https://www.youtube.com/watch?v=o9lCKRT66sE)

Course Title: Statistical Mechanics

Paper: PHY 423

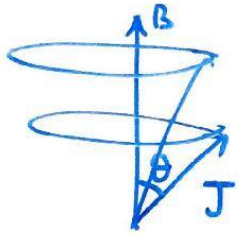
Unit: 2

Semester: 2

Name of the Teacher: Arvind Pan

Name of the Department: Physics PG

b) Quantum Mechanical approach (Para magnetism)



$\mu = g \mu_B J$; $g = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$

$E = -\vec{\mu} \cdot \vec{B}$ $\qquad \mu_B = \frac{e\hbar}{2m}$

$= -g\mu_B \vec{J} \cdot \vec{B} = -g\mu_B J \cos\theta B$

$= -g \mu_B m B$

$Q_1(\beta) = \sum_i e^{-\beta E_i}$

$= \sum_{m=-J}^{i=J} e^{\beta g \mu_B B m}$

for $\beta g \mu_B B J = x \Rightarrow \frac{x}{J} = \beta \mu_B g B$

$Q_1(\beta) = \sum_{m=-J}^{m=J} e^{\frac{x}{J} m}$ = G.P with first term e^{-x} & Common ratio $e^{x/J}$

$\frac{e^{\frac{x}{J}(-J+1)}}{e^{-x}}$: C.R

$= e^{-x} \frac{\left(e^{(2J+1)\frac{x}{J}} - 1\right)}{\left(e^{x/J} - 1\right)}$

$= \frac{e^{-x} e^{(2J+1)\frac{x}{2J}} \left(e^{(2J+1)\frac{x}{2J}} - e^{-(2J+1)x/2J}\right)}{e^{x/2J} \left(e^{x/2J} - e^{-x/2J}\right)}$

$= \frac{\sinh\left(2J+1\right)x/2J}{\sinh\left(\frac{1}{2J}\right)x}$

$$Q_N = \left[\sinh\left(\frac{2J+1}{2J}\right)x \Big/ \sinh\left(\frac{1}{2J}\right)x\right]^N \quad \left|\begin{array}{l} x = \beta(\mu_B g J)\, B \\ \frac{\partial}{\partial B} = \beta \mu_B g J \frac{\partial}{\partial x} \end{array}\right.$$

Helmholtz Free Energy

$$A = -kT \ln Q_N$$

$$= -NkT\left[\ln \sinh\left(\frac{2J+1}{2J}\right)x - \ln \sinh\left(\frac{1}{2J}\right)x\right]$$

$$M = -\left(\frac{\partial A}{\partial B}\right)_{N,T} = +NkT\beta g \mu_B J \frac{\partial}{\partial x}\left[\ln \sinh\left(\frac{2J+1}{2J}\right)x - \ln \sinh\left(\frac{1}{2J}\right)x\right]$$

$$= N g \mu_B J \left[\left(\frac{2J+1}{2J}\right)\coth\left(\frac{2J+1}{2J}\right)x - \left(\frac{1}{2J}\right)\coth\left(\frac{1}{2J}\right)x\right]$$

$$= N g \mu_B J\, B(x) \qquad B(x)$$

$$M = N g \mu_B J \cdot \frac{x}{3}\left(1+\frac{1}{J}\right) = N g \mu_B J \cdot \frac{\beta \mu_B g J B}{3}\left(\frac{J+1}{J}\right)$$

$$= \frac{N (g\mu_B)^2 J(J+1)}{3kT} B$$

$$\chi = \frac{\partial M}{\partial B} = \frac{N g^2 \mu_B^2 J(J+1)}{3kT} = \frac{C}{T} \rightarrow \text{Curie's law}$$

a) For low temp, high mag field $x \gg 1$ $B_J(x) \to 0$

$$M = N(g\mu_B J)$$

b) For high temp, low field $x \ll 1 \rightarrow B_J = \frac{x}{3}\left(1+\frac{1}{J}\right)$

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