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NAAC ACCREDITED 'A' GRADE

Topic	**: Transistor Models: h- Parameters**
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Name of the Department	**: Physics**

Transistor Models: *h- Parameters*

One of our first concerns in the sinusoidal ac analysis of transistor networks is the magnitude of the input signal. It will determine whether *small-signal* or l*arge signal* techniques should be applied. There is no set dividing line between the two, but the application—and the magnitude of the variables of interest relative to the scales of the device characteristics—will usually make it quite clear which method is appropriate.

When the signal amplitude is small the transistor may be assume to operate with reasonably linearity and we inquire into small signal linear models, which represent the operation of the transistor in the active region. The equivalent circuit for a transistor can be drawn using simple approximations by retaining its essential features, at the same time discarding its less important quantities. The parameters introduced in the models presented here are interpreted in terms of external volt ampere characteristic of the transistor. These equivalent circuits will aid to analyzing transistor circuits easily and rapidly.

The small signal operation is that in which ac input signal voltages and currents are of the order of ±10% of Q point voltages and currents.

It also be noted that at low frequency the transistor internal capacitance may be neglected.

There are three models commonly used in the small-signal ac analysis of transistor networks: the r_e model, the hybrid π model, and the hybrid equivalent model.

A model is the combination of circuit elements, properly chosen, that best approximates the actual behaviour of a semiconductor device under specific operating conditions.

The hybrid equivalent model was used in the early years before the popularity of the r_e model developed. Today there is a mix of usage depending on the level and direction of the investigation.

The r_e model has the advantage that the parameters are defined by the actual operating conditions,

Whereas

The parameters of the hybrid equivalent circuit are defined in general terms for any operating conditions.

BJT Transistor Modelling: Hybrid Model:

In hybrid model a transistor can be treated as a general two-port network as shown Fig 1. The terminal behaviour of a large class of two-port devices is specified by the terminal voltages V_i and V_0 at port 1 and 2 respectively, and currents I_i and I_o entering port 1 and 2 respectively. We may select two of the four quantities as the independent variables add express the remaining two in terms of the chosen independent variables. It should be noted that, in general, we are not to free to select the independent variables arbitrarily.

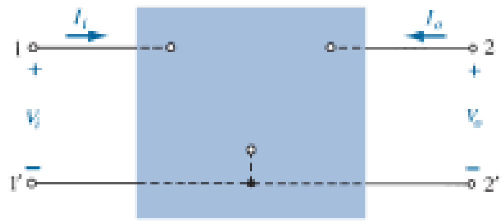


Figure: 1

Let us consider I_i and V_0 are the independent parameter foe the transistor. The following sets of equations are only one of a number of ways in which the four variables of Fig. 1 can be related.

$$V_i = h_{11}I_i + h_{12}V_0 \dots\dots\dots\dots (1.1)$$

$$I_0 = h_{21}I_i + h_{22}V_0 \dots\dots\dots\dots (1.2)$$

The parameters relating the four variables are called *h-parameters* from the word *"hybrid."* The term hybrid was chosen because the mixture of variables (*V* and *I*) in each equation results in a *"hybrid"* set of units of measurement for the h-parameters. That is the quantities h_{ij} are not dimensionally alike. Hence these quantities are termed as *hybrid* parameter.

Meaning of Hybrid parameter:

- A clearer understanding of what the various h-parameters represent and how we can determine their magnitude can be developed by isolating each and examining the resulting relationship.

If we arbitrarily set $V_0 = 0$(short circuit the output terminals) and solve for h_{11} in Eq. (1.1), the following will result:

$$h_{11} = \left[\frac{V_i}{I_i}\right]_{V_o=0} \quad Ohms \dots\dots\dots..(1.3)$$

The ratio indicates that the parameter h_{11} is an impedance parameter with the units of ohms. Since it is the ratio of the *input* voltage to the *input* current with the output terminals *shorted,* it is called the ***short-circuit input-impedance parameter.*** The subscript 11 of h_{11} defines the fact that the parameter is determined by a ratio of quantities measured at the input terminals.

- If $\boldsymbol{I_i}$ is set equal to zero by opening the input leads, the following will result for h_{12}:

$$h_{12} = \left[\frac{V_i}{V_o}\right]_{I_i=0} \quad Unitless \dots\dots\dots..(1.4)$$

The parameter h_{12}, therefore, is the ratio of the input voltage to the output voltage with the input current equal to zero. It has no units since it is a ratio of voltage levels and is called the ***open-circuit reverse transfer voltage ratio parameter***. The subscript 12 of h_{12} reveals that the parameter is a transfer quantity determined by a ratio of input to output measurements. The first integer of the subscript defines the measured quantity to appear in the numerator; the second integer defines the source of the quantity to appear in the denominator. The term *reverse* is included because the ratio is an input voltage over an output voltage rather than the reverse ratio typically of interest.

- If in Eq. (1.2) $V_o = 0$ by again shorting the output terminals, the following will result for h_{21}:

$$h_{21} = \left[\frac{I_o}{I_i}\right]_{V_o=0} \quad Unitless \ \ldots\ldots\ldots..(1.5)$$

Note that we now have the ratio of an output quantity to an input quantity. The term *forward* will now be used rather than *reverse* as indicated for h_{12}. The parameter h_{21} is the ratio of the output current to the input current with the output terminals shorted. This parameter, like h_{12}, has no units since it is the ratio of current levels. It is formally called the ***short-circuit forward transfer current ratio parameter***. The subscript 21 again indicates that it is a transfer parameter with the output quantity in the numerator and the input quantity in the denominator.

- The last parameter, h_{22}, can be found by again opening the input leads to $\boldsymbol{I_i}$ is set equal to zero and solving for h_{22} in Eq. (1.2):

$$h_{22} = \left[\frac{I_o}{V_o}\right]_{I_i=0} \quad Siemens \ \ldots\ldots\ldots..(1.6)$$

Since it is the ratio of the output current to the output voltage, it is the output conductance parameter and is measured in Siemens (S). It is called the ***open-circuit output admittance parameter***. The subscript 22 reveals that it is determined by a ratio of output quantities.

Hybrid Equivalent Circuit:

Since each term of Eq. (1.1) has the unit volt, let us apply Kirchhoff's voltage law "in reverse" to find a circuit that "fits" the equation. Performing this operation will result in the circuit of Fig.2. Since the parameter h_{11} has the unit ohm, it is represented by a resistor in Fig. 2. The quantity h_{12} is dimensionless and therefore simply appears as a multiplying factor of the "feedback" term in the input circuit.

Again since each term of Eq. (1.2) has the units of current, let us now apply Kirchhoff's current law "in reverse" to obtain the circuit of Fig. 3. Since h_{22} has the units of admittance, which for the transistor model is conductance; it is represented by the resistor symbol. Keep in mind, however, that the resistance in ohms of this resistor is equal to the reciprocal of conductance ($1/h_{22}$).

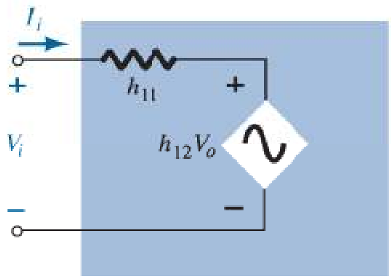


Figure: 2

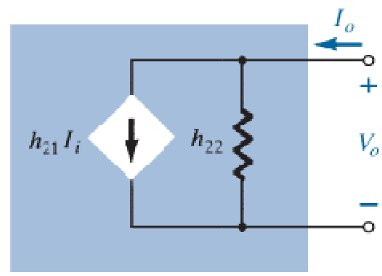


Figure: 3

The complete "ac" equivalent circuit for the basic three-terminal linear device is indicated in Fig. 4 with a new set of subscripts for the h-parameters. The choice of letters is obvious from the following listing:

$$h_{11} \rightarrow \textit{input resistance} \rightarrow h_i$$
$$h_{12} \rightarrow \textit{reverse transfer voltage ratio} \rightarrow h_r$$
$$h_{21} \rightarrow \textit{forward transfer current ratio} \rightarrow h_f$$
$$h_{22} \rightarrow \textit{output conductance} \rightarrow h_o$$

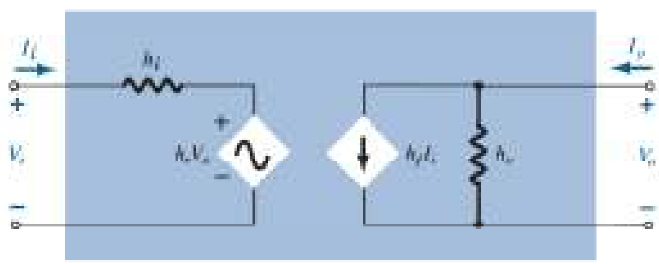


Figure: 4

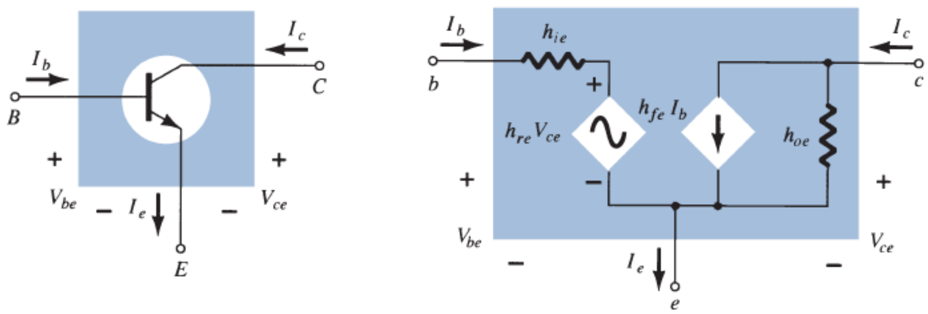
The circuit of Fig. 4 is applicable to any linear three-terminal electronic device or system with no internal independent sources. For the transistor, therefore, even though it has three basic configurations, *they are all three-terminal configurations,* so that the resulting equivalent circuit will have the same format as shown in Fig. 4. In each case, the bottom of the input and output sections of the network of Fig. 4 can be connected.

When h- parameters are applied to transistors, it is a common practice to add a second subscript to designate the type of configuration considered: e for common emitter, b for common base, and c for common collector. Thus for a common emitter (CE) configuration we may write.

$$h_{11} \rightarrow \textit{input resistance} \rightarrow h_{ie}$$
$$h_{12} \rightarrow \textit{reverse transfer voltage ratio} \rightarrow h_{re}$$
$$h_{21} \rightarrow \textit{forward transfer current ratio} \rightarrow h_{fe}$$
$$h_{22} \rightarrow \textit{output conductance} \rightarrow h_{oe}$$

For CE mode:

The hybrid equivalent network for the common-emitter configuration appears with the standard notation in Fig. 5. Note that $I_i = I_b$, $I_o = I_c$, and through an application of Kirchhoff's current law, $I_e = I_b + I_c$. The input voltage is now V_{be}, with the output voltage V_{ce}

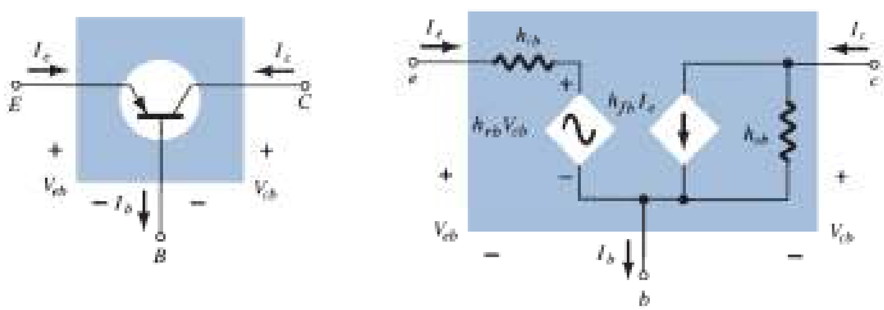


(a) (b)

Figure 5

For CB mode:

For the common-base configuration of Fig. 6, $I_i = I_e$, $I_o = I_c$ with V_{eb}=V_i and V_{cb}=V_o. The networks of Figs. 5 and 6 are applicable for *pnp* or *npn* transistors.



(a) (b)

Figure 6

Advantages of h-Parameters:

Apart from h-parameter model there are another two additional transistor equivalent circuits, called the z-parameter (Impedance parameter) and y-parameter (Admittance parameter) equivalent circuits, use either the voltage source or the current source, but not both, in the same equivalent circuit. However, h-parameter is used vary much because of its following advantages.

1. H-parameters are real numbers upto radio frequencies because at these frequencies reactive elements are absent.
2. They can be determined from transistor static characteristics curves.
3. They are convenient to use in circuit analysis and design.
4. They are easily convertible from one configuration to other.
5. They are readily supplied by manufactures.

Conversion formula for hybrid parameters:

h-Parameters	CE	CB	CC
Short circuit input impedance	h_{ie}	$h_{ib} = \frac{h_{ie}}{1+h_{fe}}$	$h_{ic} = h_{ie}$
Open-circuit reverse transfer voltage ratio	h_{re}	$h_{rb} = \frac{h_{ie}h_{oe}}{1+h_{fe}} - h_{re}$	$h_{rc} = 1$
Short-circuit forward transfer current ratio	h_{fe}	$h_{fb} = -\frac{h_{fe}}{1+h_{fe}}$	$h_{fc} = -(1+h_{fe})$
Open-circuit output admittance	h_{oe}	$h_{ob} = \frac{h_{oe}}{1+h_{fe}}$	$h_{oc} = h_{oe}$

- ***Home work:*** *Find all CB h-parameters in terms of CE h-parameters.*

Analysis of a Transistor Amplifier Circuit Using h-Parameters:

A transistor amplifier can be constructed by connecting an external load and signal source as indicated in Fig 7 and biasing the transistor properly. The two port active network of Fig 7 represents a transistor in any one of the three possible configurations. In Fig 8 we treat the general case (connection not specified) by replacing the transistor with its small signal hybrid model. The circuit used in Fig 8 is valid for any type of load, whether it be a pure resistance, an impedance, or another transistor. This is true because the transistor hybrid model was derived without any regard to the external circuit in which the transistor is incorporated. The only restriction is the requirement that the h-parameters remain substantially constant over the operating range.

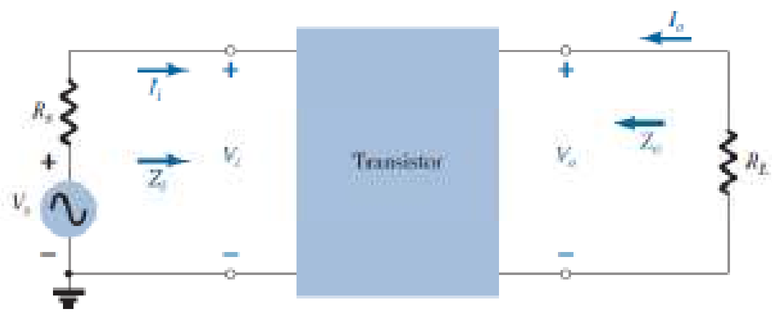


Figure 7

Assuming sinusoidally varying voltages and currents, we can proceed with the analysis of the circuit of Fig 8 using phase (sinor) notation to represent the sinusoidally varying quantities. The quantities if interests are the current gain, the input impedance, the voltage gain and the output impedance.

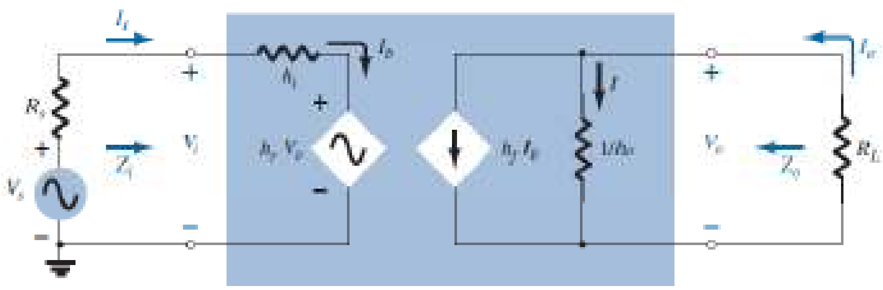


Figure 8

❖ *Current Gain:*

For a transistor amplifier, the current gain A_I is defined as the ratio of output current to the input current i.e.

$$A_i = -\frac{I_o}{I_i}$$

Applying Kirchhoff's current law to the output circuit yields

$$I_0 = h_f I_b + I = h_f I_i + \frac{V_0}{1/h_o} = h_f I_i + h_o V_0$$

Substituting $V_0 = -I_0 R_L$ gives

$$I_0 = h_f I_i - h_o I_0 R_L$$

Rewriting the above equation

$$I_0(1 + h_o R_L) = h_f I_i$$

So that

$$A_i = -\frac{I_o}{I_i} = -\frac{h_f}{(1 + h_o R_L)}$$

Note that the current gain reduces to familiar result of $A_i = -h_f$ if the factor $h_o R_L$ is sufficiently small compared to 1.

> ➢ ***Home work:*** *Find the Current gain A_{is} , taking into account the resistance R_s of the source.*

❖ *Voltage Gain:*

The ratio of output voltage V_0 to the input voltage V_i gives the voltage gain of the transistor.

Applying Kirchhoff's voltage law to the input circuit results in

$$V_i = h_i I_i + h_r V_0$$

Substituting $I_i = \frac{I_0(1+h_o R_L)}{h_f}$ and $I_0 = -\frac{V_0}{R_L}$ in above expression we get

$$V_i = -h_i \frac{V_0(1 + h_o R_L)}{h_f R_L} + h_r V_0$$

Solving for the ratio $\frac{V_0}{V_i}$ yields

$$A_V = \frac{V_i}{V_0} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L}$$

In this case, the familiar form of $A_V = \frac{-h_f R_L}{h_i}$ returns if the factor $(h_i h_o - h_f h_r) R_L$ is sufficiently small compared to h_i.

> ➢ ***Home work:*** *Find the voltage gain A_{VS}, taking into account the resistance R_s of the source.*

❖ *Input Impedance:*

The impedance we see looking into the amplifier input terminal from where input voltage measured as shown in Fig 8 is the amplifier input impedance Z_i.

Applying Kirchhoff's voltage law to the input circuit results in

$$V_i = h_i I_i + h_r V_0$$

Substituting $\quad V_0 = -I_0 R_L$

We have $\quad V_i = h_i I_i - h_r I_0 R_L$

Because $\quad A_i = -\frac{I_o}{I_i}$

$$I_o = -A_i I_i$$

So that above equation becomes

$$V_i = h_i I_i + h_r A_i I_i R_L$$

Solving the ratio $\frac{V_i}{I_i}$, we obtain

$$Z_i = \frac{V_i}{I_i} = h_i + h_r A_i R_L$$

Now again $A_i = -\frac{h_f}{(1+h_o R_L)}$

Therefore

$$Z_i = \frac{V_i}{I_i} = h_i - \frac{h_r h_f R_L}{(1 + h_o R_L)}$$

The familiar form of $Z_i = h_i$ is obtained if the second factor in the denominator $h_o R_L$ is sufficiently large.

❖*Output Impedance:*

The output impedance of an amplifier is defined to be the ratio of the output voltage to the output current with the signal V_s set to zero. For the input circuit apply Kirchoff's voltage law we may write

$$V_s = I_i(R_s + h_i) + h_r V_o$$

$$I_i = -\frac{h_r V_o}{(R_s + h_i)}$$

Applying Kirchhoff's current law to the output circuit yields

$$I_0 = h_f I_i + h_o V_0$$

Substitute I_i into the equation from the output circuit yields

$$I_0 = -\frac{h_f h_r V_o}{(R_s + h_i)} + h_o V_0$$

Therefore

$$Z_0 = \frac{V_0}{I_0} = \frac{1}{h_o - \left[\frac{h_f h_r}{(R_s + h_i)}\right]}$$

The output impedance is reduced to the familiar form $Z_0 = \frac{1}{h_0}$ for the transistor when the second factor in the denominator is sufficiently smaller than the first.