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NAAC ACCREDITED ‘A’ GRADE

Topic: Diffraction in a double slit (Fraunhofer diffraction)

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Diffraction in double slit

Double slits:

The phrase 'double slit' reminds us the famous Young's double slit experiment which is always referred to explain the interference pattern. In that experiment we treated the two sources as two points. After introduction of single slit diffraction pattern now we are aware of the status of a slit which cannot be a point source rather an array of sources. Truly speaking the pattern depends upon the nature of arrangement of the sources. In the previous section we have just finished the discussion of a linear array of sources. A slit can always be considered in this way. Certainly both the interference and diffraction take place simultaneously. Now the questions are:

1) Interference pattern was measured experimentally and the results were stated earlier in the interference section). The match between theory and the experiment was excellent. Where was the diffraction at that time?

2) Do they occupy different places?

3) Can we discuss the two things separately?

4) What will be the methodology?

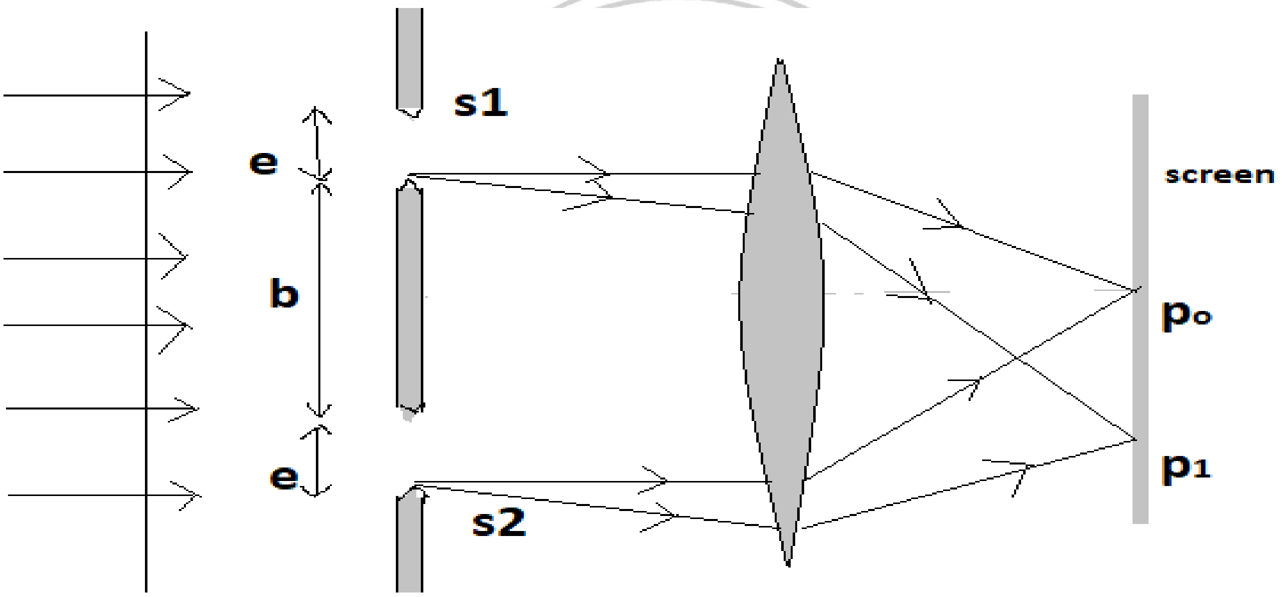


Figure 1: Ray diagram for double slit arrangement

The questions are quite legitimate. We should have some answer of the question + to start the way of action. Before that we try to realise questions 2 and 3. The answer of the question 2 is definitely no.

From the discussion of diffraction and the interference we observe the patterns vary with the deviation, θ from the normal. The full range of θ was considered for both the cases. With the question 3 we can make a plan to consider an interference of two sources of amplitude $\mathbf{A}\frac{\sin\alpha}{\alpha}$. Yes, it is a workable solution proceed along this way. We hitherto was silent about the question 1 and we hope that we would get the answer at the end of the discussion. For the time being we can make the comment that we actually observe interference pattern through diffraction pattern i.e. the pattern allowed by the diffraction pattern.

The fig 1 shows the ray diagram for the double slit arrangement. The sources are considered at the middle point of the slits. We are seeking the expression of the amplitude at an angle of deviation θ.The distance between the sources S1 and S2 is $\frac{e}{2}+\boldsymbol{b}+\frac{e}{2}$ = **e + b**. Here '**e**' is the width of the slits and '**b**' is the width of the opaque space between the two slits.

The amplitudes of the single sources are **R** = **A**$\frac{\sin\alpha}{\alpha}$, where α is related to the single slit pattern, i.e., **α =** $\frac{\pi e\sin\theta}{\lambda}$. {'d' is replaced by e }. The path difference between the two light rays coming from the sources Si and S2 is (e + b) sinθ. The corresponding phase difference can be calculated by the multiplication through the factor $\frac{2\pi}{\lambda}$. This is considered as 2β.

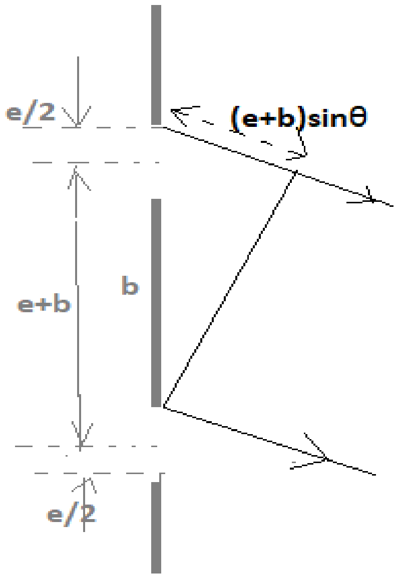


Figure 2: path difference is (e + b) sinθ

2β = (e + b) sinθ.

β ={(e + b) sinθ}/2

The equation for the two waves can be considered as **y1 = R sin(wt)** and **y2 = R sin (wt + 2β).** Therefore the resultant amplitude becomes

y = y1 + y2 = R sin (wt +2β) + R sin (wt)

$$= 2R \sin\frac{2(wt+\beta)}{2}\cos\frac{2\beta}{2}$$

=2R sin (wt +β) cos β

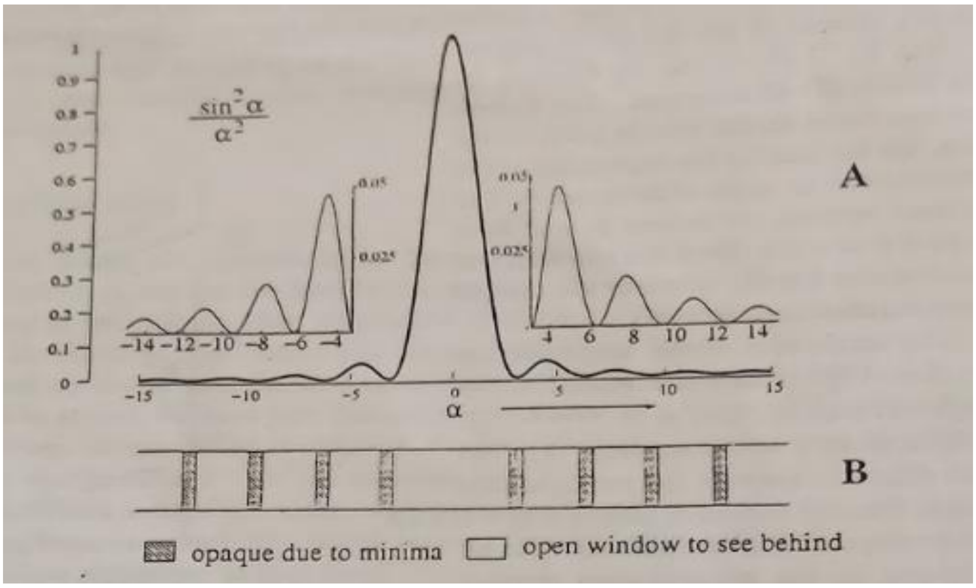
Replacing the expression of R from the single slit pattern we have

y= 2 A$\frac{\sin\alpha}{\alpha}$**.cosβ.sin (wt +β)**

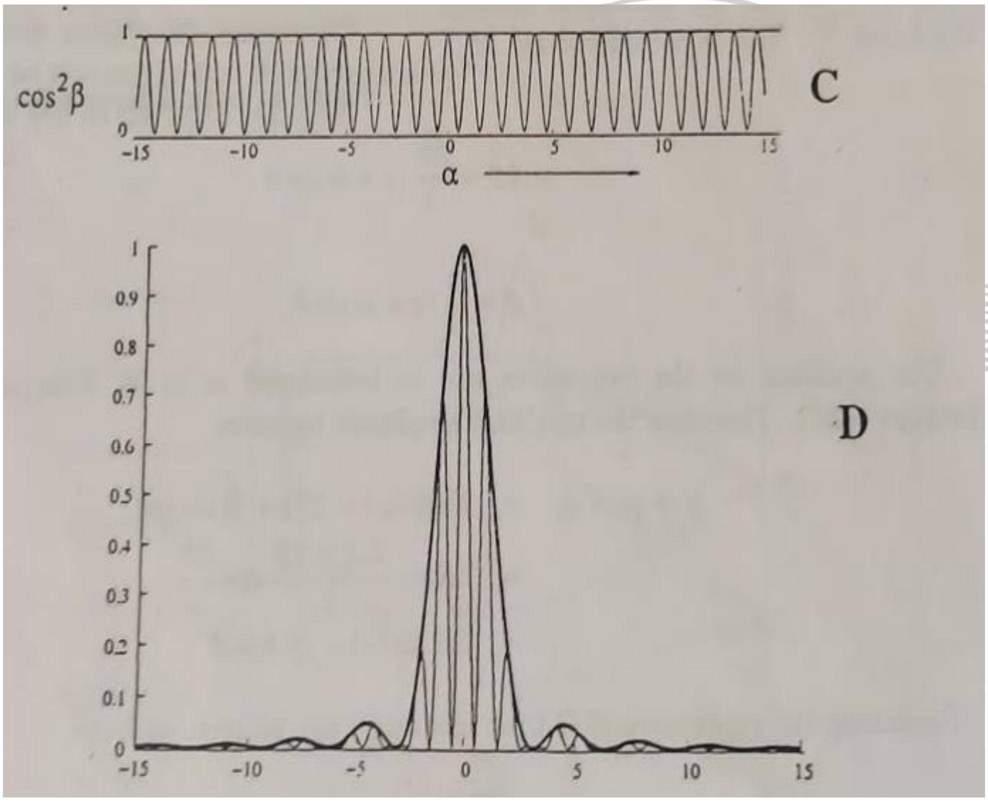
The last sin (wt +β) represent the wave form. β inside the parenthesis indicate average phase shift due to incoming waves from two slits. Rest of the part **(2.A**$\frac{\sin\alpha}{\alpha}$**.cosβ)** represent amplitude of the resultant wave.

Let us now try to understand the functions different part of the amplitude on overall intensity, graphically.

Intensity $\mathbf{I=4.A^2.(\frac{\sin\alpha}{\alpha})^2.(\cos\beta)^2}$



In this graph of double slit diffraction pattern: contribution of different parts (A) Variation of the factor $\mathbf{(\frac{\sin\alpha}{\alpha})^2}$, the secondary maxima are zoomed on either side. (B) Analogy of open window for diffraction maxima



In this graph of double slit diffraction pattern: contribution of different parts (C) interference pattern distributed uniformly which can be observed through the diffraction maxima. (D) it is the final product representing

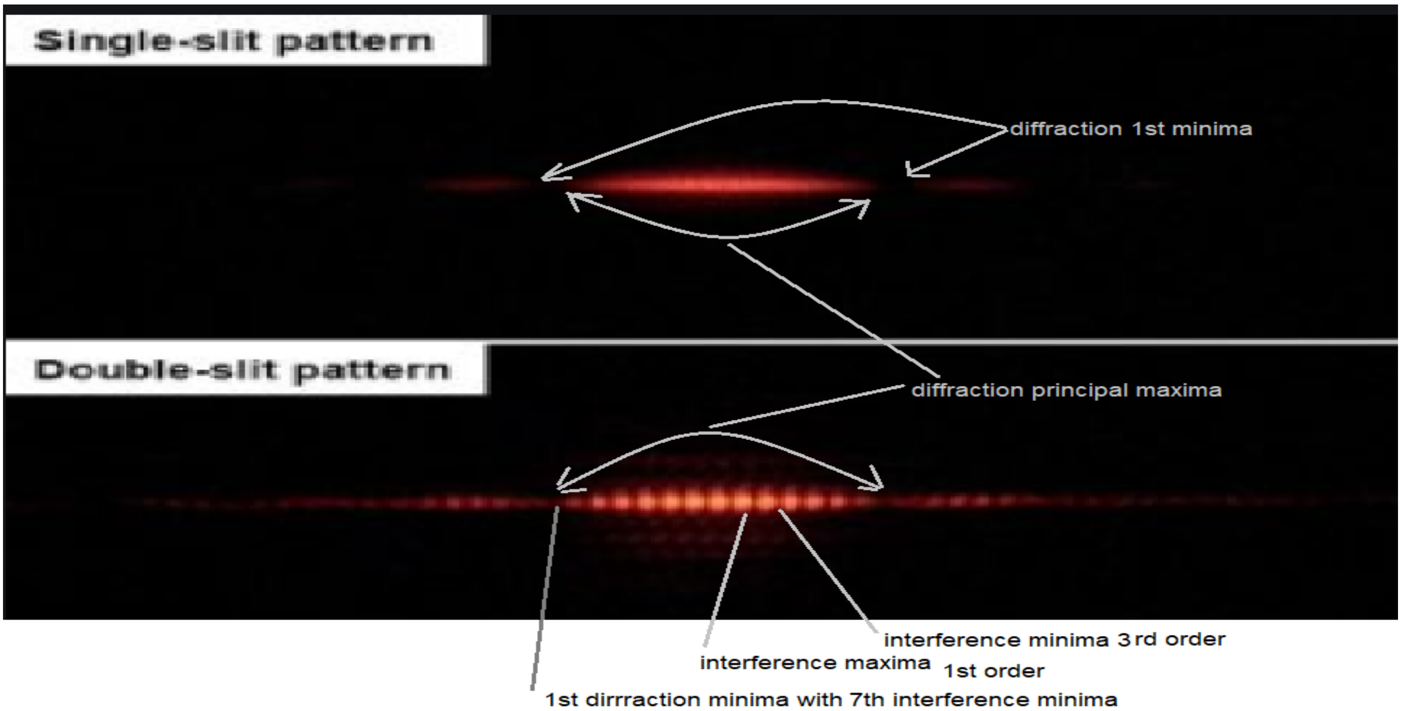
$\mathbf{I=4.A^2.(\frac{\sin\alpha}{\alpha})^2.(\cos\beta)^2}$

In this expression of intensity we found two parts which can vary with the deviation of the ray at angle θ . The firsts one is $\mathbf{(\frac{\sin\alpha}{\alpha})^2}$,with which we get familiar in the previous section. The second one is **$\cos^2\beta$.** The term **$\cos^2\beta$** produces regular maxima and minima, the interference pattern due to two slits. The phase difference or modulating factor β of this $\cos^2\beta$ variation is generated from the factor (e+b), the separation between the slits. Presence of the term like $\mathbf{A^2.(\frac{\sin\alpha}{\alpha})^2}$ produces a principal maxima and series of secondary maxima separated by minima After superposition of the two terms we will be deprived to see the interference pattern due to presence of some darkness (minima) offered by the diffraction pattern. Thus diffraction minima does not produce the dark region but also wipe out the interference pattern. Only the diffraction maxima allow us to peep through the interference pattern. Maxima provides an open window for that.

Missing order

We can comprehend the situation that some portion of the interference pattern becomes invisible due to occurrence of diffraction minima in same place. To quantify this we try to be more precise to find the order number of interference fringe which has been disappeared. The interference pattern is counted from zeroth order where path difference is exactly zero. Zeroth order therefore will be produced at the opposite side of the slits for θ = 0. It is the place for central region of principal maxima. Under no circumstance zero order vanishes. Order of the interference maxima is counted from 1 on either side of the 0th maxima or central maxima. There is no 0th order for minima. Its order starts from 1 on either side of the 0th interference maxima. The pattern is schematically shown below with bold and italic face letter for maxima and ordinary letter for minima.

7 ***6*** 6 ***5*** 5 ***4*** 4 ***3*** 3 ***2*** 2 ***1*** 1 ***0*** 1 ***1*** 2 ***2*** 3 ***3*** 4 ***4*** 5 ***5*** 6 ***6*** 7



In the whole range of θ we get some values of θ where both the condition for diffraction minima and condition for interference maxima are satisfied. As we mentioned earlier the position of the diffraction minima are exactly same with the single slit diffraction pattern. Here slit width of each slit is e. So we recall expression to describe the diffraction minima as **e.sin θ = mλ**.---------(1)

We are interested about the order number and the pattern is symmetrical. So we drop out the negative sign.

From the expression (**2.A$\frac{\sin\alpha}{\alpha}$.cosβ**) we get the interference pattern is taken care by the factor cosβ. Maxima of interference pattern is therefore is denoted by **cosβ** = 1. To get that β must be integral multiple of π. Some values obviously produce -1 but intensity is square of the factor **cosβ**. Let β = sπ. From the expression of β we get

$$\beta = \frac{\pi}{\lambda}(e+b)\sin\theta = s\pi$$

(e + b) $\sin\theta$ = sλ.---------------------------------(2)

Sth order of interference maxima will be missing (under mth order of diffraction minima) of the equations (2) and (1) are satisfied simultaneously. The place of occurrence is denoted by the angle ϑ which is same for both the equations.

We are lucky enough that both the expressions contain sine of the θ and when θ is same sinθ is also same. Therefore from the two equations we have

$$\frac{(e+b)\sin\theta}{e\sin\theta}=\frac{s\lambda}{m\lambda}$$

$$1+\frac{b}{e}=\frac{s}{m} \text{ -------------(3)}$$

This is the relation between the order number (s) of interference pattern which is missing and the order number of diffraction minima (m) due to which interference maxima is missing. And it is clearly depends upon the relative width of the slit (e) and the opaque space (b) between them.

We may try to find out the relation between α and β the controlling parameters of diffraction and interference pattern respectively.

$$\alpha = \frac{\pi}{\lambda}e\sin\theta$$

$$\beta = \frac{\pi}{\lambda}(e+b)\sin\theta$$

$$\frac{\beta}{\alpha}=\frac{(e+b)}{e}=1+\frac{b}{e}$$

So that it is equal to the same ratio $\frac{s}{m}$.

To realise this missing order let us try with some different ratios of b and e.

1) width of the opaque space is same as the width of the slit: (b= e) Using expression (3) we get

$$1+\frac{b}{e}=1+1=2=\frac{s}{m}$$

or **s=2m**

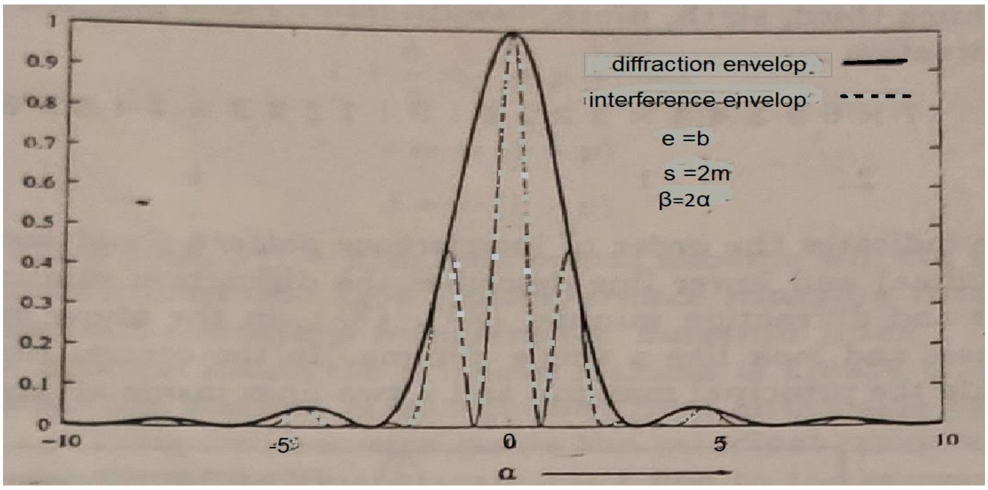
so **β=2α**

For m= 1, 2, 3, 4, . . . s= 2, 4, 6, 8. . . . That means under the first, second, third diffraction minima second, fourth, sixth interference maxima will disappear. Let us describe the situation

7 x 6 ***5*** 5 ***x*** 4 ***3*** 3 ***x*** 2 ***1*** 1 ***0*** 1 ***1*** 2 ***x*** 3 ***3*** 4 ***x*** 5 ***5*** 6 X 7

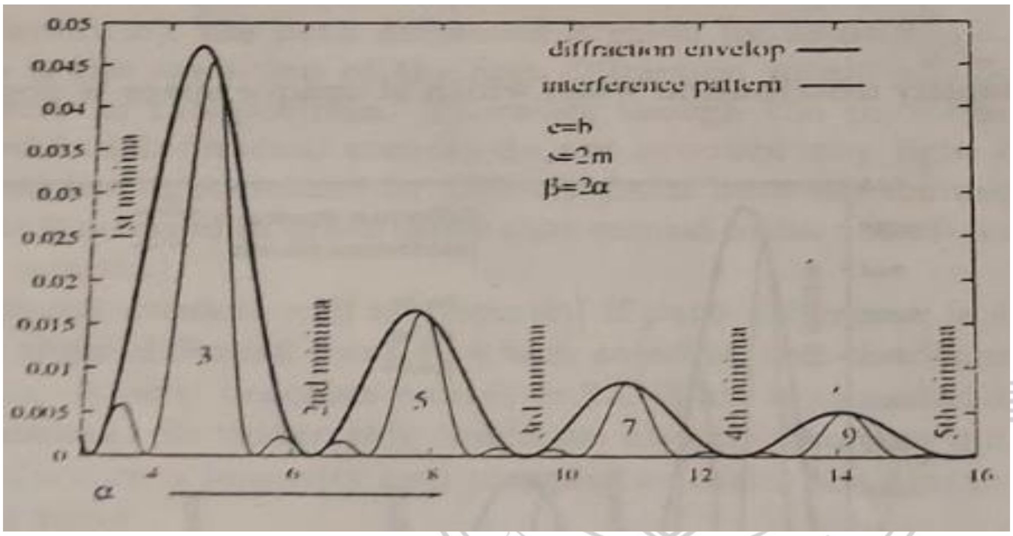
3 2 1 1 2 3

Upper line indicates the order of interference pattern (bold & italic face font: maxima; ordinary font: minima) and lower line describes the diffraction minima. Darkness has no identity. The combinations of interference and diffraction minima $(\frac{2\ 3}{1})$,$(\frac{4\ 5}{2})$,$(\frac{6\ 7}{3})$ in the above representation produce a single darkness and look like a single minima. In the central region we have three lines (maxima) inside the principal maxima and a inside all the secondary maxima. See the fig below



This Figure shows intensity distribution when slit width is same as the width of the opaque space .

The intensity of the principal maxima is so high the other secondary maxima become negligible. We zoomed the other part excluding the principal maxima in the fig below. The figure is started from first minima. Inside all the secondary diffraction maxima one interference maxima is present. The orders of these interference maxima are depicted with them. Diffraction minima is formed at the place where the interference maxima of order 2, 4, 6, 8, 10 would form. Small signature of such missing maxima are found inside the diffraction envelop.



This Figure is same as before fig but just zoomed on the secondary maxima

2) Width of the opaque space is double of the width of the slit: (b = 2e) Using expression (3) we get

$$1 + \frac{b}{e} = 1 + 2 = 3 = \frac{s}{m}$$

S = 3m

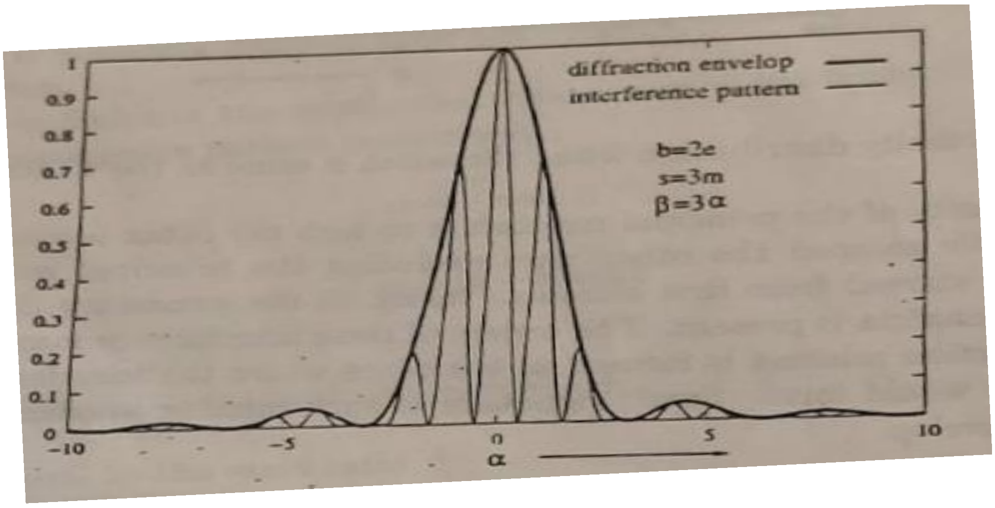
So β=3α

For m= 1, 2, 3, 4 . . . .s=3. 6. 9,12, . . . That means under the first, second, third diffraction minima third, sixth, ninth, twelfth interference maxima will disappear. Let us describe the situation

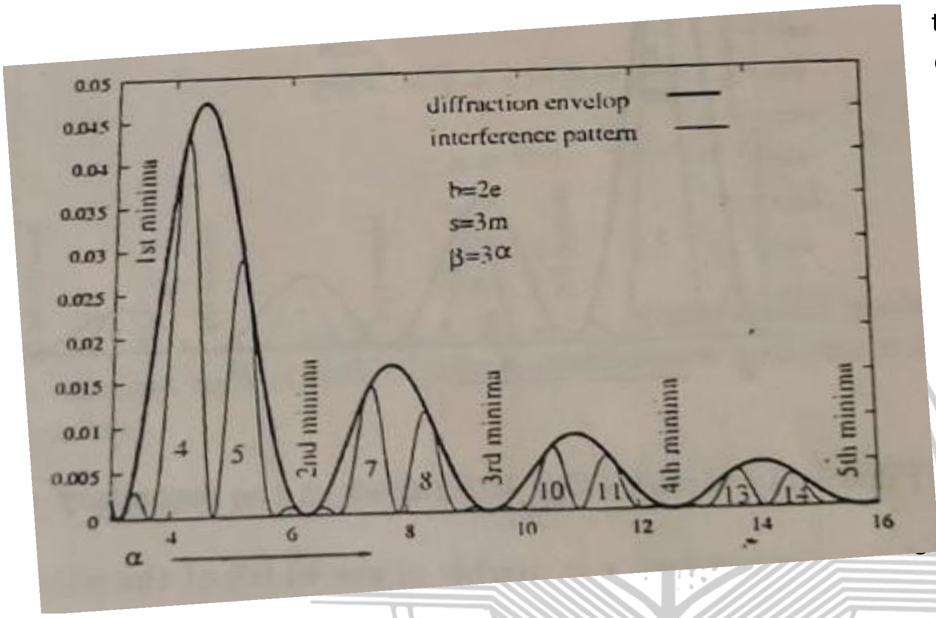
7 x 6 **5** 5 **4** 4 x 3 **2** 2 **1** 1 **0** 1 **1** 2 **2** 3 x **4** 4 5 **5** 6 X 7

2 1 1 2

Upper line indicates the order of interference pattern (bold and big face font: maxima: ordinary font: minima) and lower line describes the diffraction minima. The combinations of interference and diffraction minima ($\frac{3\ 4}{1}$), ($\frac{6\ 7}{2}$), in the above representation produce a single darkness and look like a single minima. In the central region we have five (maxima) inside the principal maxima and a two lines inside all the secondary maxima.



this figure shows intensity distribution when width of opaque space is double of the slit width.



this figure is same as previous fig but only zoomed on the secondary maxima'

Let us generalise the result.

3) width of the opaque space is p[th] times the width of the slit width: (**b=p.e**)

Let us consider b=pe. We can write the equation (3) as

$$1+\frac{b}{e} = 1+p = \frac{s}{m}$$

= m(1+p) So **β=α(1+p)**

So (1+p)th , 2 (1+p)th, 3(1 + p)th interference maxima will not exist.

Principal maxima contains 2p + 1 **interference marima lines**.

Secondary maxima contain p **interference maxima lines**.

The number of maxima inside the principal maxima is always odd in number.the same order of interference maxima is extinguished from either side by the first minima say s. Therefore s-1 maxima remains inside the principal maxima in one side. On the other side the number of maxima is again s -1 due to the symmetric arrangement. Central interference fringe has the order number 0. So the total number of maxima inside the principal maxima is 1+2 (s - 1) which is definitely odd. The number of interference maxima inside the secondary diffraction maxima depends on the relative width of the opaque space the integer p.

All these are represented in mathematical form. The missing order can also be explained physically. Let us consider the case where slit width exactly matches with the width of the opaque space; i.e., **e = b**. If we consider the interfering sources are present at the center of the two slits, then the distance between them in the given case is simply **2e**. For a point where we expect a second order interference maxima the path difference between them should be **2λ**. If the position for second interference maxima is denoted by angular deviation O from the direction of the incident ray (defined by perpendicular to the incident waveform), the path difference is given by **2e.sinθ**. i.e., **2e sin θ** = **2λ**. Or **esin θ** = **λ**. This is the condition of the first diffraction minima. So the contribution from each slit is zero in this position. Therefore though the condition for second order maxima is satisfied but individual sources do not produce any light due to destructive interference between the light created by different parts between themselves we always get zero intensity. This is referred in other sense that second order interference is missing (due to first diffraction minima).

Similarly the fourth maxima will be occurred if path difference is **4λ** i.e**., $2e.\sin\theta_1$, = 4λ**. **θ_1**, is another angle different from **θ**, which satisfies the condition for second order interference maxima. Now it becomes **$e \sin\theta_1 = 2\lambda$**. This is the condition for the second order diffraction minima. So under this condition also the sources individually refuse to contribute and produce zero intensity and conspire to make the fourth interference order to be in the missing state.

Let us consider again a separate case where slit width is half of the width of the opaque space. i.e., **$b = 2e$**. Therefore distance between two interfering sources is **$e+b=3e$**. If **θ_1** is the condition where third order interference maxima is occurred then the path difference at this position between the rays coming from the two sources is **$3e \sin\theta = 3\lambda$**. This can be written as **$e \sin\theta = \lambda$**. Without any hesitation one can declare that the intensity of this place should be zero due to non-cooperation of individual slit with destructive interference which causes the occurrence of the 1st minima. Similarly the 6th ,9th ,…. Maxima will be disappeared under the influence of 2nd , 3rd … diffraction minima.
