VIVEKANANDA COLLEGE
THAKURPUKUR
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NAAC ACCREDITED ‘A’ GRADE

Topic: Dielectric Materials- Class-2-10/04/2020-Electric dipole_ field_torque on dipole

Course Title: Electricity and magnetism

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Name of the Department: Physics

Dielectric materials

Class-2

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10/04/2020

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Syllabus

1. Electric dipole moment, electric potential and field due to an electric dipole, force and torque on a dipole.

2. Electric fields inside matter, Electric polarization, bound charges, displacement density vector, relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$. Gauss's theorem in dielectrics, linear dielectric medium, electric susceptibility and permittivity, electrostatic boundary conditions for $\vec{E}$ and $\vec{D}$.

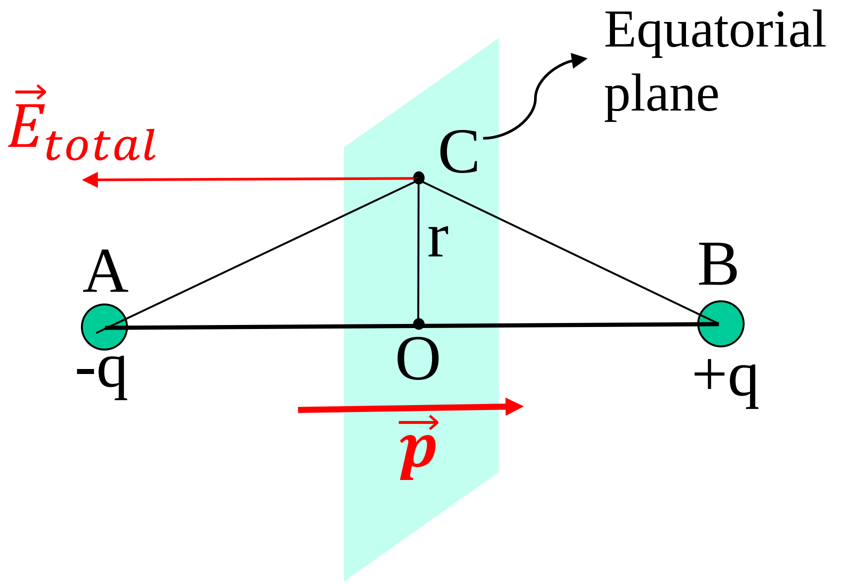
Today's topic: <u>Electric field due to dipole at any general point, dipole in a uniform electric field (torque on dipole)</u>

Electric field due to dipole at any general point:

- We know that for far away point,

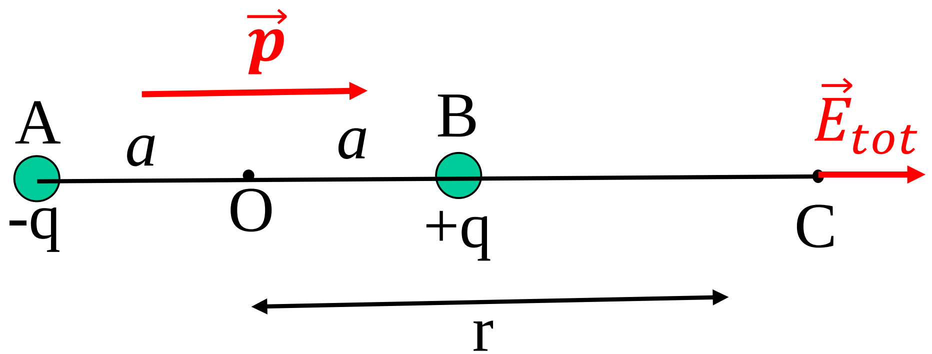
field at point on equatorial plane of the dipole is

$$\vec{E}_{dipole} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$



- and field at point on axis of the dipole is

$$\vec{E}_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$



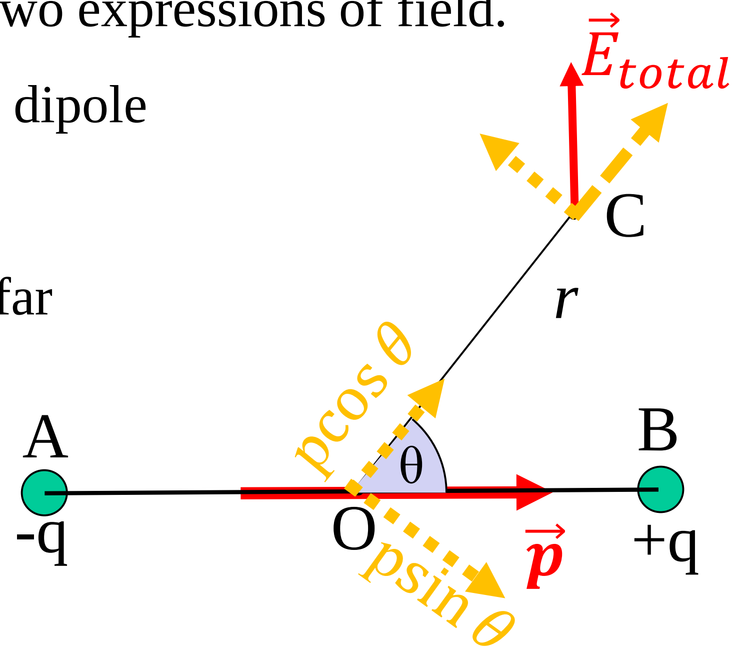
Electric field due to dipole at any general point

Now, for any general point which is neither on axis nor on equatorial plane we can derive the expression of the field using the above two expressions of field.

Let C is a point at distance *r* from the centre of the dipole making an angle θ with the axis of the dipole.

Let dipole length is 2*a* such that *r* >> *a i.e*, C is a far away point.

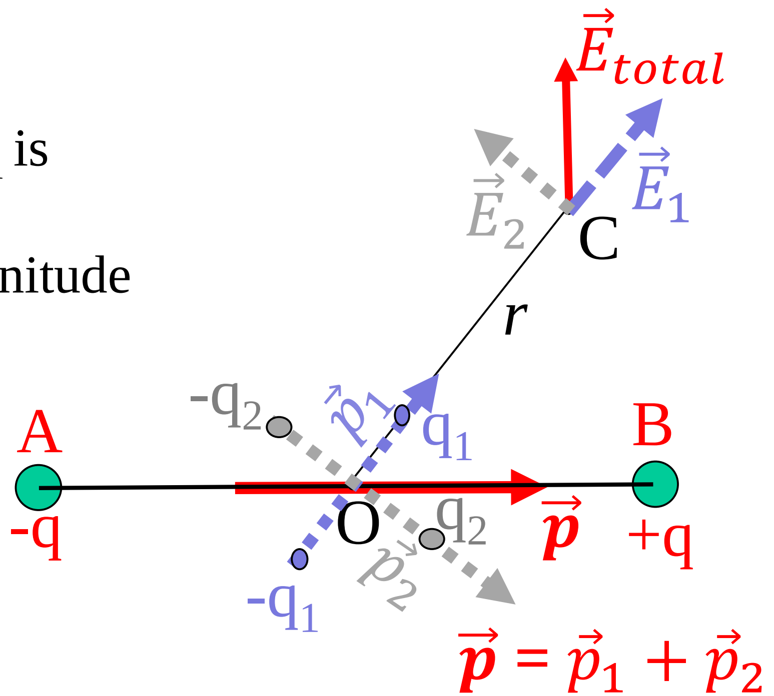
To calculate the electric field at C we shall decompose the dipole moment $\vec{p}$ into two components such that the point C becomes axial point for one component and equatorial point for the other component.



From the above figure, the components pcos θ and psin θ are along and perpendicular to the line OC. So, we may imagine that the point C is on the axis of a dipole p_1 having dipole moment of magnitude equal to pcos θ.

Electric field due to dipole at any general point

Similarly, we can imagine that the point C is on the equatorial plane of a dipole p_2 having dipole moment of magnitude equal to psin θ. So the electric field E_1 at C due to dipole p_1 will be the same as for any axial point. Similarly, the electric field E_2 at C due to dipole p_2 will be the same as any equatorial point.



So, for the far away point C the field due to dipole p_1 is

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}_1}{r^3}$$ where $\vec{p}_1$ = pcos $\theta\ \hat{n}_1$, p is the magnitude of dipole moment of original dipole.

Similarly,

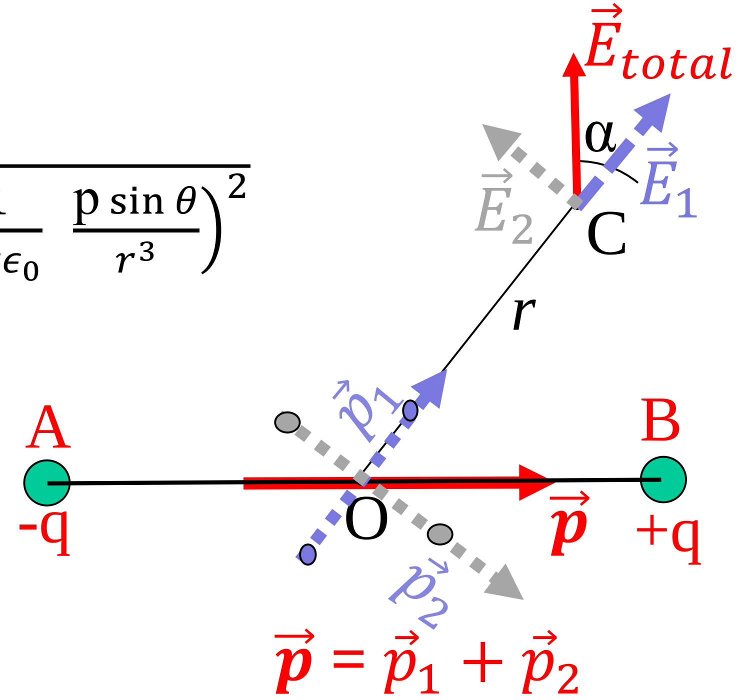
$$\vec{E}_2 = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}_2}{r^3}$$ where $\vec{p}_2$ = psin $\theta\ \hat{n}_2$

Thus total electric field at the far away point C is given by vector sum as,

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{2\text{p}\cos\theta}{r^3}\hat{n}_1 - \frac{1}{4\pi\epsilon_0}\frac{\text{p}\sin\theta}{r^3}\hat{n}_2$$

Electric field due to dipole at any general point

- The magnitude of $\vec{E}_{total}$ can be derived as



$$|\vec{E}_{total}| = \sqrt{\left({E_1}^2 + {E_2}^2\right)} = \sqrt{\left(\frac{1}{4\pi\epsilon_0}\frac{2\text{p}\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\epsilon_0}\frac{\text{p}\sin\theta}{r^3}\right)^2}$$

$$= \sqrt{\left(\frac{1}{4\pi\epsilon_0}\frac{\text{p}}{r^3}\right)^2 \{(2\cos\theta)^2 + (\sin\theta)^2\}}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\frac{\text{p}}{r^3}\right)\sqrt{\{3(\cos\theta)^2 + 1\}}$$

- Direction of $\vec{E}_{total}$ can be derived as

$$\tan\alpha = \frac{|\vec{E}_2|}{|\vec{E}_1|} = \frac{1}{2}\frac{\sin\theta}{\cos\theta} = \frac{1}{2}\tan\theta \qquad \left[|\vec{E}_1| = \frac{1}{4\pi\epsilon_0}\frac{2\text{p}\cos\theta}{r^3} \text{ and } |\vec{E}_2| = \frac{1}{4\pi\epsilon_0}\frac{\text{p}\sin\theta}{r^3}\right]$$

Where α is the angle of $\vec{E}_{total}$ with the line OC.

Electric field due to dipole at any general point

So, for any general far away $(r>>a)$ point C the magnitude of field due to dipole is

$$|\vec{E}_{total}| = =\left(\frac{1}{4\pi\epsilon_0}\frac{\text{p}}{r^3}\right)\sqrt{\{3(\cos\theta)^2+1\}}$$

and its direction makes angle α with the position vector $(\vec{r})$ of the point C from the centre of the dipole where

$$\tan\alpha = \frac{1}{2}\tan\theta$$

θ being the angle the position vector $\vec{r}$ makes with the axis of the dipole at the centre.

Electric dipole at uniform electric field

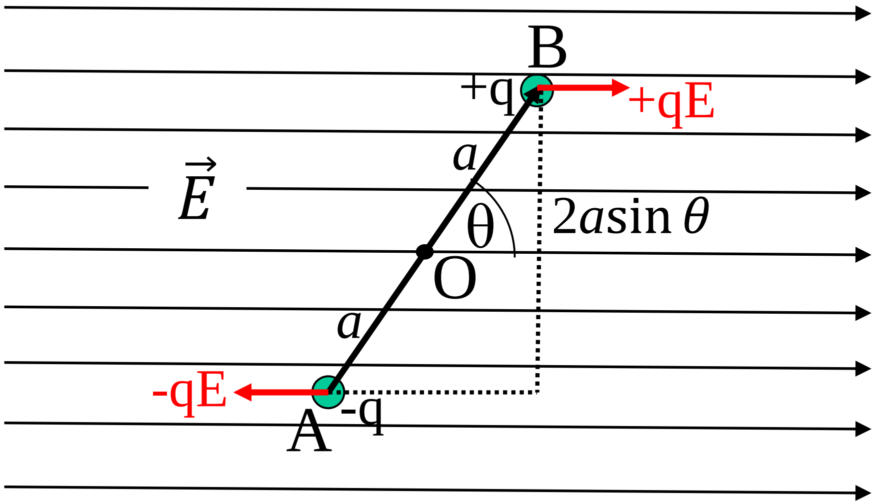
In the previous classes we have studied the field at any point due to an electric dipole i.e, if a dipole is kept in any place how much field will be generated by the dipole at any point in space.

Now, we will study if the dipole is placed in an external electric field what will be the effect of the field on the dipole.

First, let the dipole is kept in a uniform electric field $\vec{E}$.

Uniform electric field $\vec{E}$**:** Field has same magnitude and same direction at each point. So, represented by **equispaced** parallel lines of forces pointed towards the same direction.

Electric dipole at a uniform electric field



In the uniform field E the force acting on the charge +q of the dipole is qE in the direction of the field and the force on charge -q is -qE in a direction opposite to the field.

Since the field is uniform, these two equal and opposite forces acting on the dipole will cancel out each other and hence net force acting on the dipole is zero.

But these two equal and opposite forces acting on two different points will form a couple and the dipole will experience a torque.

This torque will try to rotate the dipole to align it along the electric field.

The total torque acting on the dipole about the centre of the dipole O is

$\vec{\tau} = \overrightarrow{OA} \times \left(-q\vec{E}\right) + \overrightarrow{OB} \times \left(q\vec{E}\right)$ [torque $\vec{\tau} = \vec{r} \times \vec{F}$]

Electric dipole at a uniform electric field

- The torque is perpendicular to the screen/page and is directed inward.
- The magnitude of the torque is

$$|\vec{\tau}| = |\overrightarrow{OA}|\,|(-q\vec{E})|\sin\theta + |\overrightarrow{OB}|\,|(q\vec{E})|\sin\theta = a\,qE\sin\theta + a\,qE\sin\theta$$

Or, $|\vec{\tau}| = qE.2a\sin\theta = pE\sin\theta$

- Where $2aq = p$, the dipole moment of the dipole and θ is the angle dipole moment p makes with the direction of field E.
- The torque can be expressed as

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

- The magnitude of torque is, $\boxed{|\vec{\tau}| = pE\sin\theta}$

  So the torque is maximum when θ is 90° and the torque is zero when θ is 0°.
- So the torque tends to align the dipole along the field such that the angle θ becomes 0° and hence the total torque acting on the dipole becomes zero.

Summary

- Electric field due to electric dipole at any general point
- Electric dipole at a uniform electric field (torque on dipole).

Next topic: *dipole in non-uniform field, dipole-dipole interaction*