VIVEKANANDA COLLEGE
THAKURPUKUR
KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: Dielectric Materials- Class-3-12/04/2020-Electric dipole_ force on dipole_potential energy

Course Title: Electricity and magnetism

Paper: CC3

Unit: N.A.

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Name of the Department: Physics

Dielectric materials

Class-3

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Syllabus

1. Electric dipole moment, electric potential and field due to an electric dipole, force and torque on a dipole.

2. Electric fields inside matter, Electric polarization, bound charges, displacement density vector, relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$. Gauss's theorem in dielectrics, linear dielectric medium, electric susceptibility and permittivity, electrostatic boundary conditions for $\vec{E}$ and $\vec{D}$.

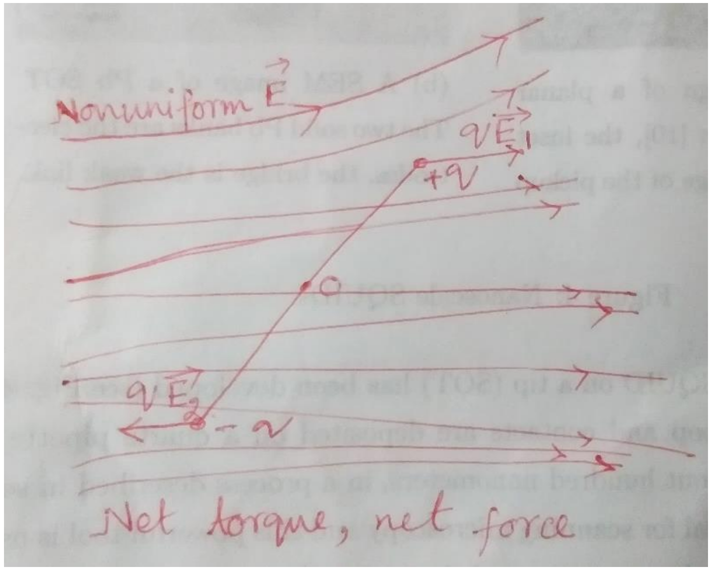
Today's topic: Electric dipole in a nonuniform electric field, potential energy of a dipole

When a dipole is kept in a uniform electric field $\vec{E}$ there is **no net force** acting on the dipole. But it experiences a torque which tries to align the dipole along the electric field.

The torque is $\vec{\tau} = \vec{p} \times \vec{E}$ with magnitude $|\vec{\tau}| = pE \sin\theta$

where *p is* the dipole moment of the dipole and θ is the angle dipole moment *p* makes with the direction of field E.

Electric dipole in a nonuniform electric field:



Let the dipole of charge –q, +q with dipole moment p is placed in a nonuniform field $\vec{E}$. Since the field is nonuniform, field at each point of space will be different.

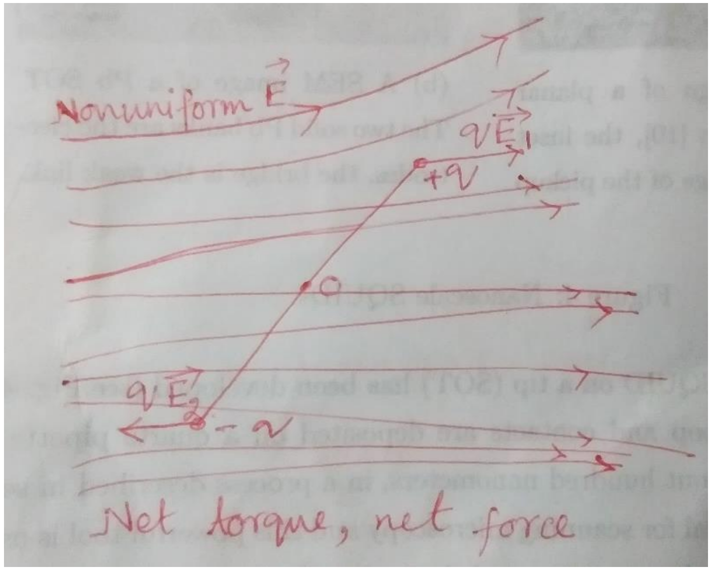
So, in this case the electric field at point of charge -q will be different from that for point of charge +q.

Electric dipole at a nonuniform electric field

Let the field on +q is $\vec{E}_1$ and hence force is q$\vec{E}_1$. Similarly, if the field on -q is $\vec{E}_2$ the force acting is -q$\vec{E}_2$.

Since the field is nonuniform i.e, $\vec{E}_1$ and $\vec{E}_2$ are not equal, the net force in this case will be nonzero. In addition, the two forces acting at different points in opposite direction will form a couple and hence the dipole will experience a torque.

So, in this case the dipole will experience both torque and force. The torque will try to rotate the dipole along the field and the force will give a translational motion to the dipole.



Net force on the dipole is

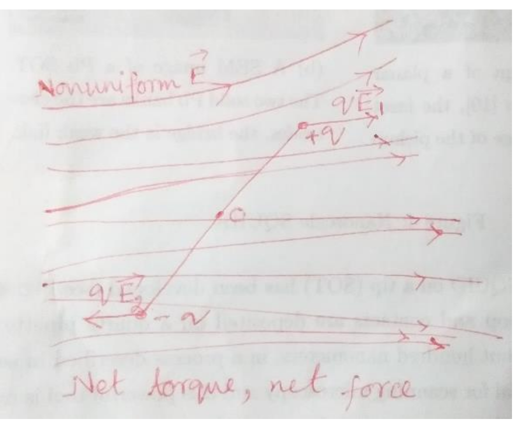
$$\vec{F} = \vec{F}_1 + \vec{F}_2 = q\vec{E}_1 - q\vec{E}_2 = q(\vec{E}_1 - \vec{E}_2)$$

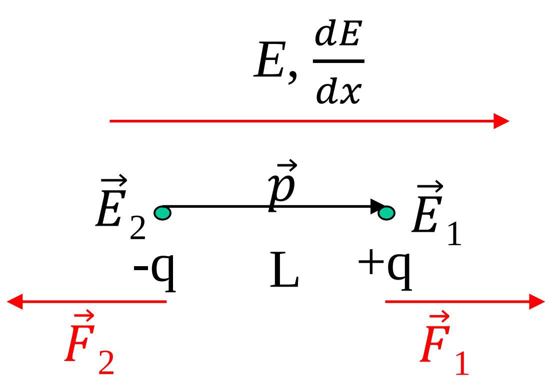
Electric dipole at a nonuniform electric field

If a dipole of length L is placed on x axis and the field changes at a constant rate $\frac{dE}{dx}$ along x axis then the magnitude of the force on the dipole is

$$F = q\,\frac{dE}{dx}L = p\,\frac{dE}{dx}$$

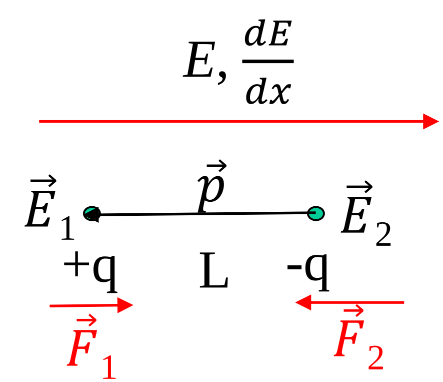
Where $p=qL$ is the dipole moment of the dipole. And the direction will be along the increase or decrease of the field depending on the orientation of the dipole.





$\vec{E}_1 > \vec{E}_2$

Force will be along the direction of increase of field



$\vec{E}_2 > \vec{E}_1$

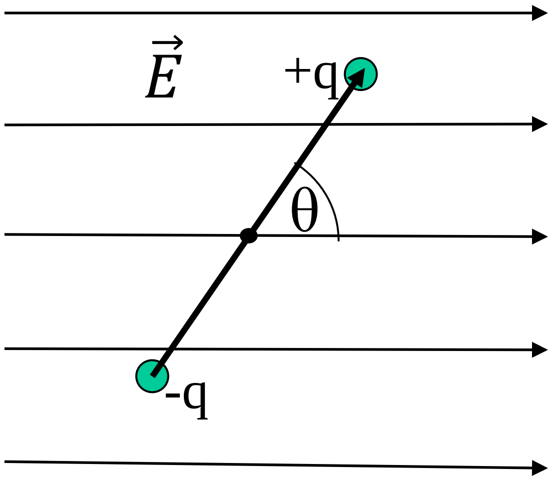
Force will be along the direction of decrease of field

Work done to rotate a dipole in a uniform electric field:

In a uniform electric field of magnitude E the torque acting on a dipole of moment p is

$\tau = \mathrm{pE}\,sin\,\theta$

where θ is the angle the dipole makes with electric field.



The work done by the torque τ to rotate the dipole by

an angle $d\theta$ is $dW = \tau d\theta$ [$dW = Fdx$]

To rotate from angle θ_1 to θ_2 the total work done is

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE\,sin\,\theta\,d\theta = pE\int_{\theta_1}^{\theta_2} sin\,\theta\,d\theta$$

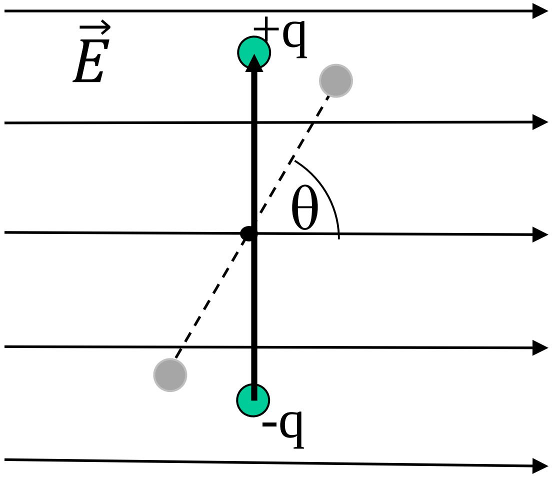
$$\boxed{W = pE[\cos\theta_1 - \cos\theta_2]}$$

Now, if the dipole is rotated from 0° to θ then the work done is

$$W = pE[1 - \cos\theta]$$

Potential energy of a dipole in a uniform electric field:

In a uniform electric field of magnitude E the work done by the torque τ to rotate the dipole from θ_1 to θ_2 is



$$W = pE[\cos\theta_1 - \cos\theta_2]$$

this work done is stored in the dipole as potential energy.

If the dipole is initially at an angle 90° with the electric field then the work done to rotate to angle θ is

$$W = -pE\cos\theta$$

This is the potential energy U.

Thus, $U = -pE\cos\theta$ Or, $U = -\vec{p}.\vec{E}$

For θ= 90° U= 0. For θ= 0° i.e, dipole moment is parallel to electric field, $U = -pE$, minimum energy, means stable equilibrium position. And θ= 180° i.e, dipole moment is antiparallel to electric field, U= pE, maximum, unstable equilibrium position.

Next topic: Electric fields inside matter, Electric polarization, bound charges, displacement density vector, relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$.