VIVEKANANDA COLLEGE
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NAAC ACCREDITED 'A' GRADE

Topic: Differential Scattering Cross-section

Course Title: **SCATTERING THEORY**

Paper: Quantum Mechanics II

Unit: PHY 422

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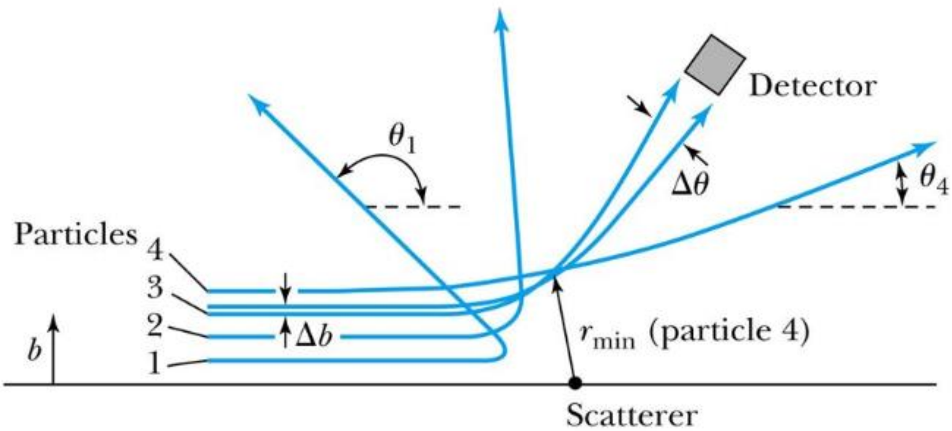
Classical Scattering Theory:

In a scattering experiment, one observers the collisions between a beam of incident particles and a target material. The total number of collisions over the duration of the experiment is proportional to the total number of incident particles and to the number of target particles per unit area in the path of the beam. In these experiments, one counts the collision products that come out of the target. After scattering, those particles that do not interact with the target continue their motion (undisturbed) in the forward direction, but those that interact with the target get scattered (deflected) at some angle as depicted in Figure below.

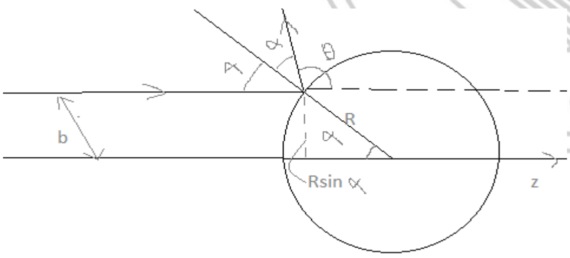


Let us consider a particle incident towards some scattering centre (p-4) with an energy E and impact parameter 'b' emerges at some angle θ. For classical scattering the target is very hard and heavy such that there is no recoil speed and due to azimuthal symmetry the trajectory remain in a plane. In classical scattering impact parameter is given & we have to find out the scattering angle θ.

Impact patameter(b):-

It can be defined as the perpendicular distance to the closest approach if the projectile were undeflected. According to the figure if 'b' increase, the scattering angle will decrease.

Relation between 'b' and θ for hard sphere scattering:



$\theta = \pi - 2\alpha$

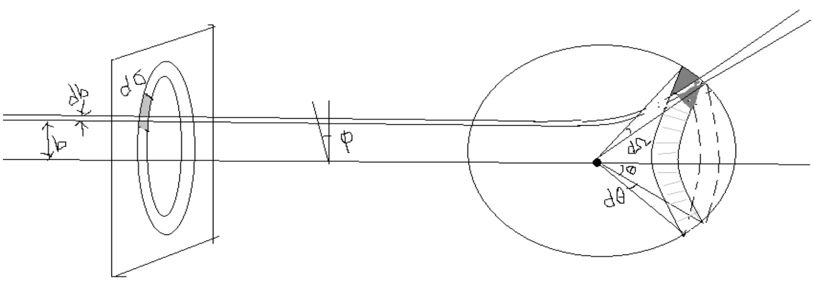
$\alpha = \pi/2 - \theta/2$

$b = R \sin\alpha = R \sin(\pi/2 - \theta/2) = R \cos(\theta/2)$

So $\theta = 2 \cos^{-1}(b/R)$ for $b <= R$

$= 0$ for $b >= R$

Differential scattering cross-section:-



Particle incident within an infinitesimal patch of cross-section area dσ will scatter through a corresponding infinitesimal solid angle dΩ.

Increasing dσ make dΩ increase. i.e. dσ ∝ dΩ or $\frac{d\sigma}{d\Omega}$=D(θ).

This factor D(θ) or $\frac{d\sigma}{d\Omega}$ is named as differential scattering cross-section.

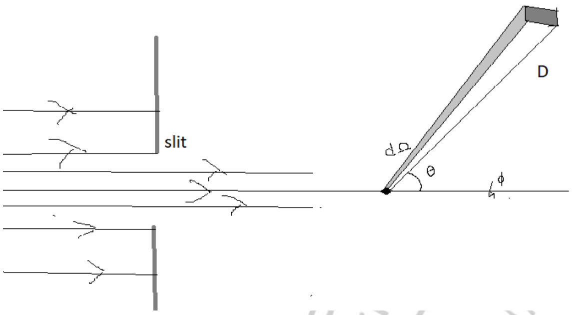
Now σ depends on b. If b increase θ & dΩ will decrease.

$$d\sigma = b.db.d\phi \text{ and } d\Omega = \sin\theta.d\theta.d\phi$$

So $$\frac{d\sigma}{d\Omega} = \frac{b.db.d\phi}{\sin\theta.d\theta.d\varphi} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|.$$

Physical meaning of $\frac{d\sigma}{d\Omega}$:

To count the no. of particles in a scattering experiment a detector D is placed outside the path of the incident beam. If the detector subtends a solid angle dΩ at the scattering centre in the direction (θ,ϕ) , then the no. of particle dN.dΩ entering into the detector per unit time can be measured.



The incident flux F, defined as the no. of particles crossing a unit area placed normal to the incident direction per unit time. If we assumed that the beam is uniform and the particle density is very low such that possibility of interactions between the beam particles themselves can be ignored. The no. of particles entering into the area dσ = F.dσ.

The no. of particles scattered through dΩ = dN.dΩ.

As $$dN = F.d\sigma. = f.D(\theta).d\Omega$$

then $$D(\theta) = \frac{dN}{F.d\sigma} = \frac{d\sigma}{d\Omega}.$$

Total scattering cross-section $\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega = \int_0^{2\pi} d\Phi \int_0^{\pi} \sin\theta \frac{d\sigma}{d\Omega} = 2\pi \int_{-1}^{1} d(\cos\theta) \frac{d\sigma}{d\Omega}$.

For hard sphere scattering:

we have $b = R\cos(\theta/2)$. $\left|\left(\frac{db}{d\theta}\right)\right| = R(\sin\theta/2)\cdot\frac{1}{2}$

Then $\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\left(\frac{db}{d\theta}\right)\right|$

$= \frac{b}{\sin\theta}\cdot R\left(\sin\frac{\theta}{2}\right)\cdot\frac{1}{2}$

$= \frac{Rb\sin(\theta/2)}{2\sin(\theta/2)\cdot\cos(\theta/2)}\cdot\frac{1}{2}$

$\frac{d\sigma}{d\Omega} = \frac{Rb}{4}\frac{1}{\cos(\theta/2)} = \frac{R}{4}\frac{1}{\cos(\theta/2)}\cdot R\cos(\theta/2)$

$\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$

$\sigma_{tot} = \int \frac{d\sigma}{d\Omega}\cdot d\Omega = \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)\cdot\frac{R^2}{4}$

$= 2\pi\cdot 2\cdot\frac{R^2}{4} = \pi R^2$