VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC accredited 'A' GRADE

Topic: Magnetic Properties of Matter

Course Title: Electricity and Magnetism

Paper:PHS-A-CC-2-3-TH

Unit: 2

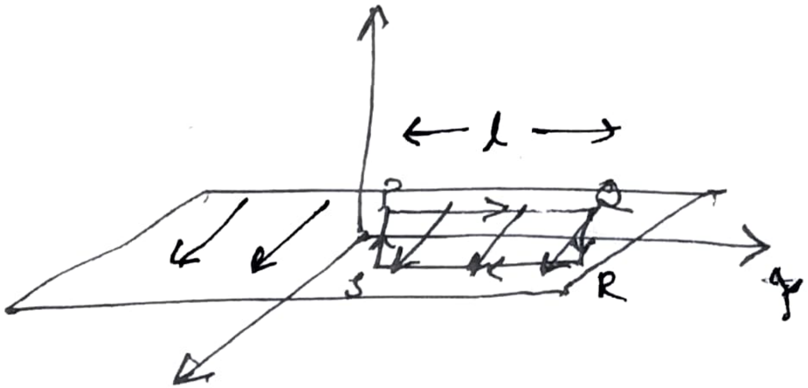
Semester: 2

Name of the Teacher:Arvind Pan

Name of the Department:Physics

Infinite Sheet:

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$



$$\int_{PQ} B \cdot dl + \int_{QR} B \cdot dl + \int_{RS} B \cdot dl + \int_{SP} \vec{B} \cdot \vec{dl} = \mu_0 K l$$

$K \rightarrow$ Surface current

$$Bl + 0 + Bl + 0 = \mu_0 K l$$

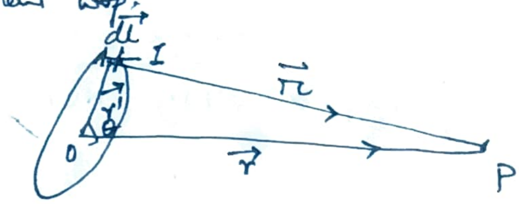
$$2Bl = \mu_0 K l$$

$$B = \frac{\mu_0 K}{2}$$

$$\vec{B} = \frac{\mu_0 K}{2}(-\hat{j}) \text{ for } z > 0$$

$$= \frac{\mu_0 K}{2}\hat{j} \text{ for } z < 0$$

Magnetic field due to an arbitrary current loop.



$$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \oint \frac{I\,\vec{dl}}{\mathscr{r}}$$

$$\mathscr{r} = |\vec{\mathscr{r}}| = \sqrt{(\vec{r}-\vec{r}').(\vec{r}-\vec{r}')} = \sqrt{r^2 + r'^2 - 2\vec{r}.\vec{r}'}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left(1 + \left(\frac{r'}{r}\right)^2 - 2\frac{\vec{r}.\vec{r}'}{r^2}\right)^{1/2}$$

$$= \frac{1}{r}\left(1 - \frac{2\vec{r}.\vec{r}'}{r^2} + \left(\frac{r'}{r}\right)^2\right)^{-1/2}$$

$$\approx \frac{1}{r}\left(1 + \frac{\vec{r}.\vec{r}'}{r^2} + O\left(\left(\frac{r'}{r}\right)^2\right)\right) \text{ for } |r| >> |r|'$$

$$\vec{dl} = d\vec{r}'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \oint I\,d\vec{r}' \times \frac{1}{r}\left(1 + \frac{\vec{r}.\vec{r}'}{r^2} + \ldots\right)$$

$$= \frac{\mu_0 I}{4\pi}\left[\oint \frac{d\vec{r}'}{r} + \oint \frac{d\vec{r}'(\vec{r}.\vec{r}')}{r^3} + \ldots\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{1}{r}\underbrace{\oint d\vec{r}'}_{0} + \frac{1}{r^3}\oint d\vec{r}'(\vec{r}.\vec{r}')\right]$$

$$\vec{r} \times (d\vec{r}' \times \vec{r}') = d\vec{r}'(\vec{r}.\vec{r}') - \vec{r}'(\vec{r}.d\vec{r}')$$

$$d[\vec{r}'(\vec{r}.\vec{r}')] = d\vec{r}'(\vec{r}.\vec{r}') + \vec{r}'(\vec{r}.d\vec{r}')$$

$$\vec{r} \times (d\vec{r}' \times \vec{r}') + d[\vec{r}'(\vec{r}.\vec{r}')] = 2\,d\vec{r}'(\vec{r}.\vec{r}')$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \times \frac{1}{2}\left[\frac{1}{r^3}\underbrace{\oint d[\vec{r}'(\vec{r}.\vec{r}')]}_{0} + \frac{1}{r^3}\oint \vec{r} \times (d\vec{r}' \times \vec{r}')\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{2}\left[\frac{1}{r^3}\oint \vec{r} \times (d\vec{r}' \times \vec{r}')\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{2r^3}\oint(\vec{r}' \times d\vec{r}') \times \vec{r}$$

$$\vec{m} = \frac{I}{2}\oint \vec{r}' \times d\vec{r}' \rightarrow \text{magnetic moment}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \vec{r}}{r^3}$$

$\frac{1}{2}\oint \vec{r}' \times d\vec{r}'$ = Area of the loop.

$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0}{4\pi} \nabla \times \left(\vec{m} \times \vec{r}/r^3 \right)$$

$$\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A})$$

$$\nabla \times \left(\vec{m} \times \vec{r}/r^3 \right) = \frac{\nabla \times (\vec{m} \times \vec{r})}{r^3} + \nabla\left(\frac{1}{r^3}\right) \times (\vec{m} \times \vec{r})$$

$$\nabla \times (\vec{m} \times \vec{r}) = (\vec{r} \cdot \nabla)\vec{m} - (\vec{m} \cdot \nabla)\vec{r} + \vec{m}(\nabla \cdot \vec{r}) - \vec{r}(\nabla \cdot \vec{m})$$

$(\vec{r} \cdot \nabla)\vec{m} = 0$, $\nabla \cdot \vec{m} = 0$

$\vec{m} =$ const.

$$= -\vec{m} + 3\vec{m} = 2\vec{m}$$

$$\nabla\left(\frac{1}{r^3}\right) = \frac{-3\vec{r}}{r^5}$$

$$\nabla \times \left(\vec{m} \times \vec{r}/r^3 \right) = \frac{2\vec{m}}{r^3} - \frac{3}{r^5} \vec{r} \times (\vec{m} \times \vec{r})$$

$$= \frac{2\vec{m}}{r^3} - \frac{3}{r^5} \left[\vec{m} r^2 - \vec{r}(\vec{m} \cdot \vec{r}) \right]$$

$$= \frac{2\vec{m}}{r^3} - \frac{3\vec{m}}{r^3} + \frac{3\vec{r}(\vec{m} \cdot \vec{r})}{r^5}$$

$$= \frac{-\vec{m}}{r^3} + \frac{3\vec{r}(\vec{m} \cdot \vec{r})}{r^5}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \left[\frac{3\vec{r}(\vec{m} \cdot \vec{r})}{r^5} - \frac{\vec{m}}{r^3} \right]$$