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NAAC ACCREDITED ‘A’ GRADE

Topic: NZ graph and Liquid Drop model
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NZ graph and Liquid Drop model

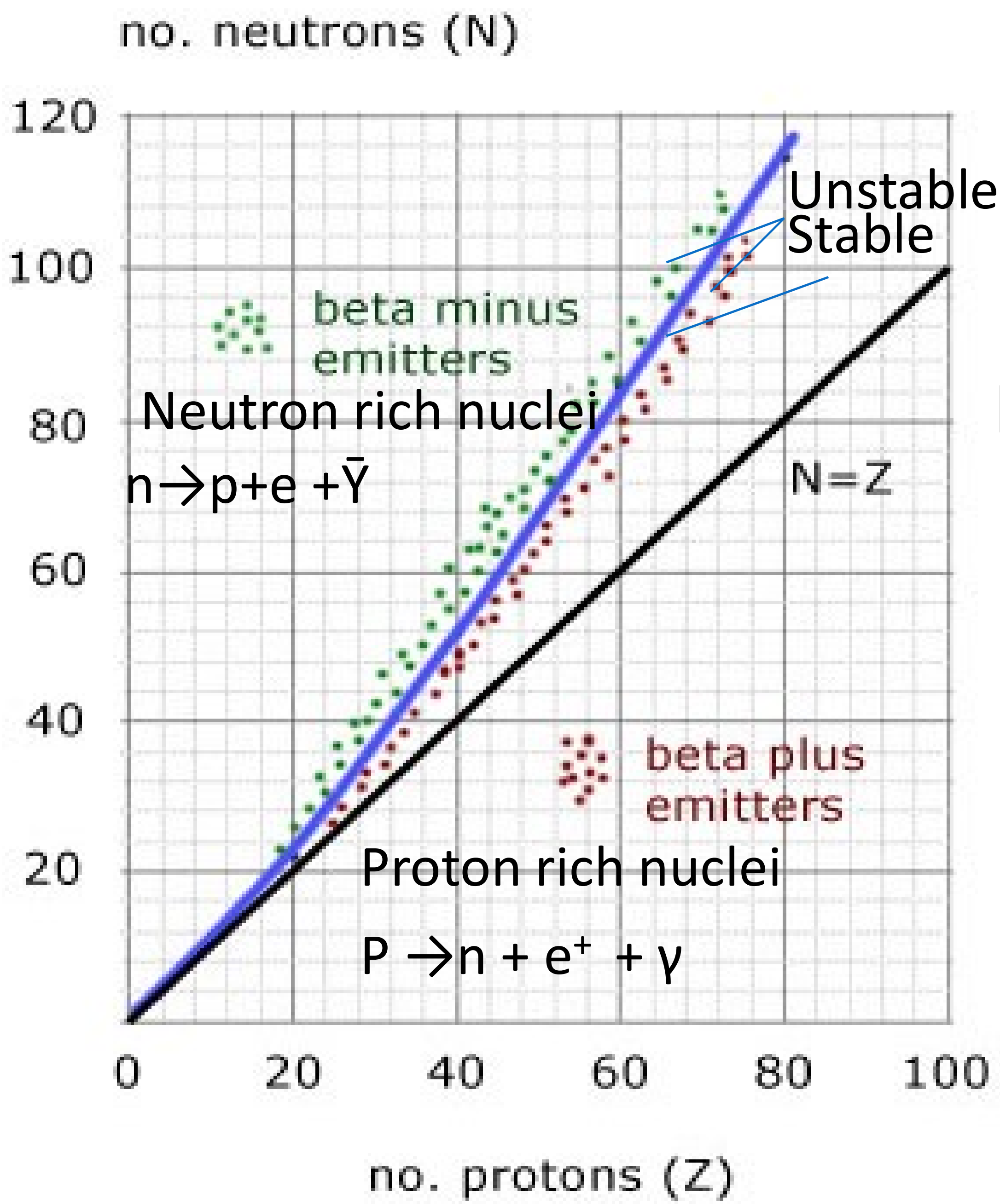
- Stability Curve or NZ graph
- Liquid drop model
- Semi-empirical mass formula

- **Stability Curve or NZ graph**

A simple way of understanding nuclear information is the N-Z plot. Some time it is also called the stability curve.

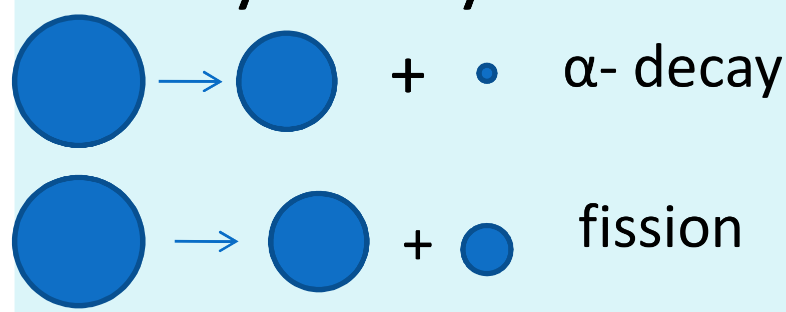
Figure represents that the curve gradually shifted from the line N/Z = 1, for low Z, this is because to counteract the coulomb repulsion of increased protons more neutrons are needed to attain stability.

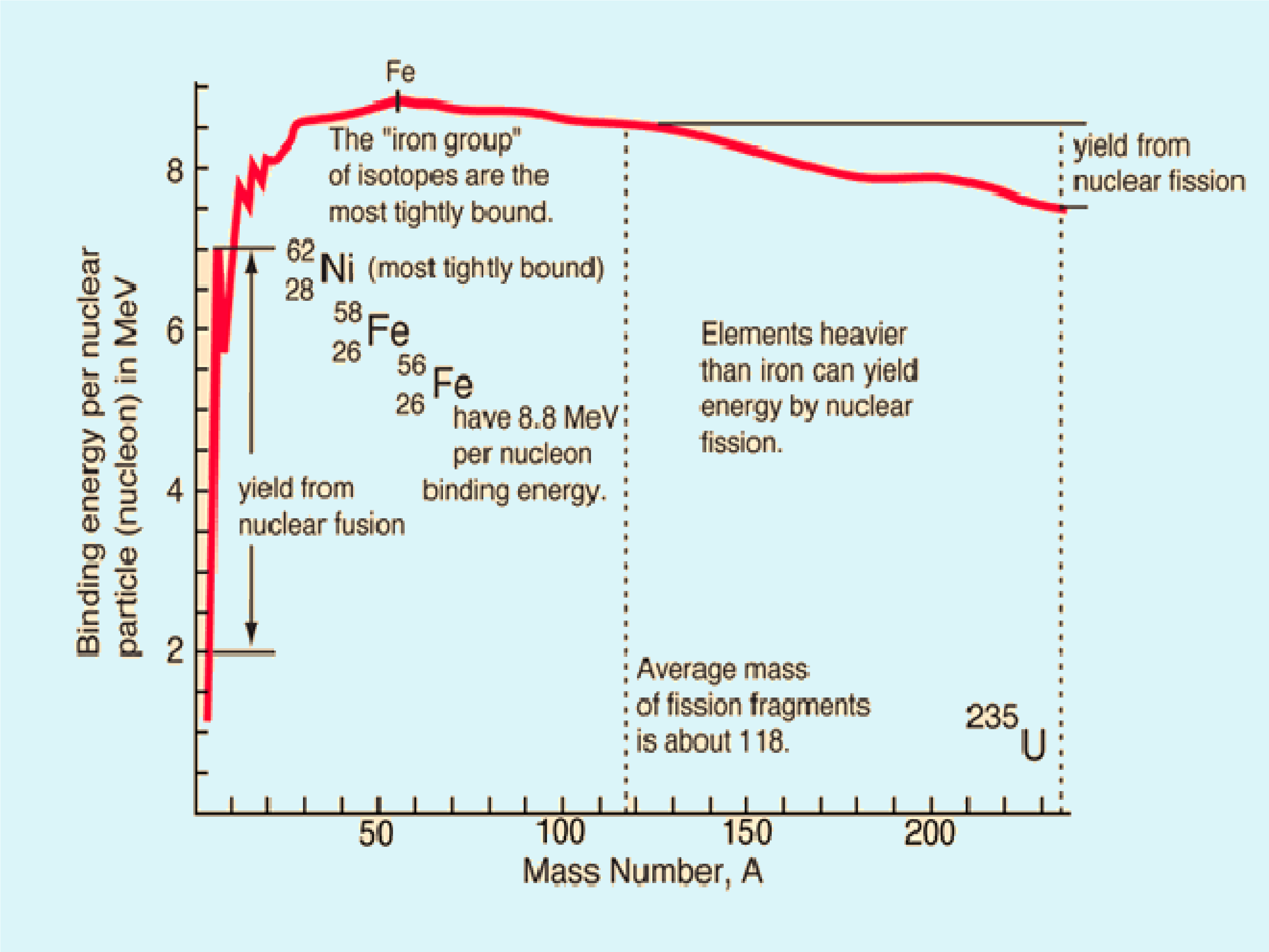
The nuclei lying above and below the stability curve are radioactive and spontaneously decay in such a way that the product nuclei lies on the stability curve.



Stable state is the minimum energy state of the nucleus.

For very heavy nuclei




Fe
The "iron group" of isotopes are the most tightly bound.
yield from nuclear fission
8
6
4
2
Binding energy per nuclear particle (nucleon) in MeV
62 28 Ni (most tightly bound)
58 26 Fe
56 26 Fe
have 8.8 MeV per nucleon binding energy.
Elements heavier than iron can yield energy by nuclear fission.
yield from nuclear fusion
Average mass of fission fragments is about 118.
235 U
50
100
150
200
Mass Number, A

Liquid drop model:

Nuclear binding energy is the energy that holds the nucleons together.

Now from where does this binding energy come?

This binding energy comes from the attractive force of nucleons.

If each nucleons interacts with all the nucleons in the nucleus, the number of interacting pairs should be

$$ {}^{A}_{2}C = \frac{A(A-1)}{2} $$

Where A is the total number of Nucleons. As A is large

The number of pairs would be $\frac{A^2}{2}$

i.e. $BE \alpha \frac{A^2}{2}$ or $\frac{BE}{A} \alpha A$ But the experimental evidence does not supports this.

The experimental evidence demand that all pairs are not contributing to the nuclear BE.

Which demands that the nuclear force is short range.

i.e. the nucleon interacts with a limited number of nucleons in its immediate vicinity.

Let each nucleon is surrounded by 'n' number of nucleons in its immediate vicinity.

Then for A number of nucleon the number of pair would be

$$\frac{An}{2}$$

Which gives that BE α A , i.e. $\frac{BE}{A} \alpha \, Constant$

Supports the experimental evidence

- Therefore, like the molecule of liquid drop nucleon in the nucleus also exerts an attractive force upon a group of nucleons in its immediate neighbourhood.
- The attractive force near the nuclear surface is similar to the force of surface tension on the surface of the liquid drop.
- Like liquid drop the density of nuclear matter is constant and independent of A i.e. independent of nuclear volume.
- In nuclear reaction different types of particles like neutrons, protons, deuterons, α-particles are emitted which is analogous to the emission of molecules from the liquid drop during evaporation.
- The internal energy of the nucleus is analogous to the heat energy within the liquid drop.
- The formation of a short lived compound nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to liquid phase in case of the liquid drop.

Observing all these similarities between the drop of a liquid and a nucleus Niels Bohr and F. Kalcar in 1936 first proposed the liquid drop model of the nucleus.

Bethe-Weizsacker formula or Semi-empirical mass formula:

The semi-empirical formula for the nuclear masses gives a connection bet^n the theory of nuclear matter with experimental information and is based on the liquid drop model of nucleus.

If M(A,Z) be the atomic mass of the isotope of an element X of atomic number Z and mass number A, then we can write

$$M(A,Z) = ZM_P + NM_n - \frac{BE}{C^2}$$

The binding energy BE can be expressed as the sum of a number of terms as given below:

1. Volume energy term:

According to Liquid drop model the binding energy is proportional to mass number A.

Again A α R^3 i.e. Corresponds to volume of nucleus

Hence we can write the first term in the expression of binding Energy as

$$E_v = a_v A$$

where a_v is a constant, called the volume coefficient

2. Surface energy term:

Since the nucleus is assumed to be identical to a spherical liquid drop of radius $R = r_0 A^{\frac{1}{3}}$

The nucleons at the surface of the nucleus are not completely surrounded by other nucleons.

The total binding energy is thus reduced due to nucleons on the surface.

This correction due to surface energy, E_S is proportional to the surface area of the nucleus i.e. $4\pi R^2$

Again $R \,\alpha\, A^{\frac{1}{3}}$ or $E_S \,\alpha\, A^{\frac{2}{3}}$

$$E_s = a_S A^{\frac{2}{3}}$$

a_S Is called the surface coefficient.

3. Coulomb energy term:

The third term, E_C is the Coulomb electrostatic repulsion between the protons, in the nucleus. Since each charged particle repulses all other charged particles, so this term is proportional to the possible number of combinations for a given proton number Z,

i.e.

$$^{Z}C_2 = \frac{Z(Z-1)}{2}$$

The energy of interaction between the protons being inversely Proportional to the distance of separation R, the energy associated with Coulomb repulsion is

$$E_C = K\frac{Z(Z-1)}{R} = K\frac{Z(Z-1)}{R_0 A^{\frac{1}{3}}} = a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

Since the repulsive effect dilutes the binding energy, it appears as a negative quantity in the semi-empirical mass formula.

4. Asymmetry energy term:

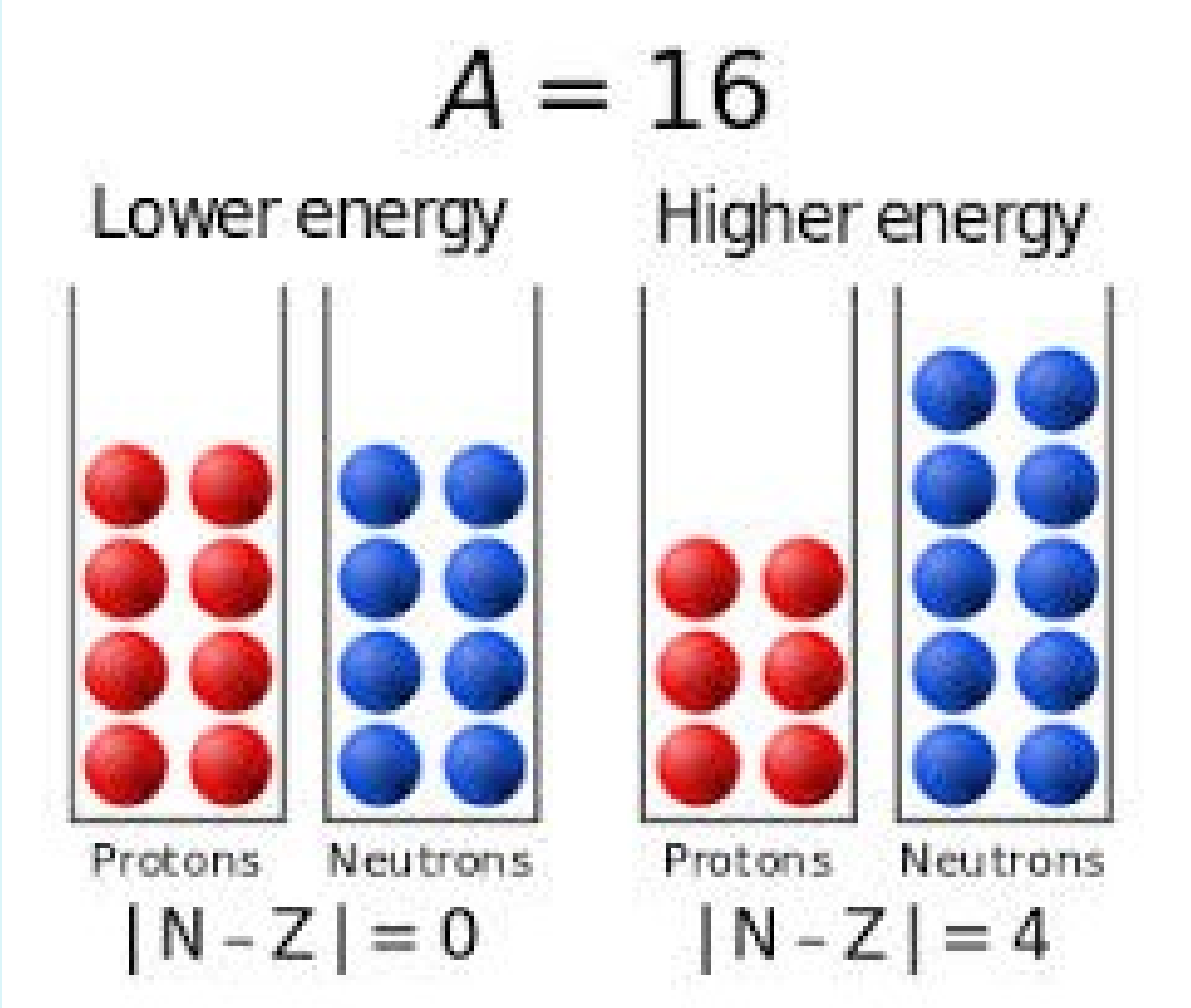
The fourth term originates from the asymmetry between the number of protons and neutrons in the nucleus. For light nuclei the number of protons is almost equal to that of neutrons: N=Z. As 'A' increases, the symmetry of proton and neutron number Becomes lost and the number of neutrons exceeds that of protons to maintain nuclear stability. This excess neutron (N-Z) is the measure of the asymmetry and it decreases the stability or B.E of the medium or heavy nuclei.

The asymmetry energy term is directly proportional to

(i) the excess neutron (N-Z) or (A-2Z)

(ii) the fraction of nuclear volume in which the excess neutrons are present

As the nuclear volume is proportional to A, the fractional volume of the nucleus in which excess neutrons are present will be proportional to (N-Z)/A



A = 16
Lower energy
Higher energy
Protons
Neutrons
|N – Z| = 0
Protons
Neutrons
|N – Z| = 4

Therefore, Asymmetry energy term $E_a \propto (N - Z)$

$$\propto \frac{(N - Z)}{A}$$

$$\therefore E_a \propto \frac{(N - Z)^2}{A}$$

$$E_a = a_n \frac{(A - 2Z)^2}{A}$$

where a_n is a constant, called the asymmetry co-efficient.

Pairing energy term:

Nuclear data indicate that nuclei with even Z and even N are most stable, where as nuclei having odd Z and odd N are least stable, and nuclei with odd N and even Z or even N and odd Z lie in between. Each of the protons and neutrons having spin ½ from pairs with parallel and anti-parallel spins in even N -even Z type nuclei giving them a stable configuration.

But in odd Z- odd N type nuclei, one unpaired proton and one unpaired neutron are left to make the nuclei less stable. So the pairing of spins increases the B.E of even Z- even N type nuclei and decreases it in odd Z- Odd N nuclei. Thus, the correction term E_P of pairing energy which is proportional to $A^{-\frac{3}{4}}$

$$E_P = \frac{a_P}{A^{\frac{3}{4}}}$$

This relation was determined empirically by Fermi.

No correction term is necessary if A is odd i.e. for A odd, $E_P = 0$

A	Z	N	No. of stable nuclei	E_P
Odd	even	odd	55	0
Odd	odd	even	50	0
Even	even	even	165	+ Ve
Even	odd	odd	4	- Ve

Odd – odd 4 nuclei $^{2}_{1}H_1$, $^{6}_{3}Li_3$, $^{10}_{5}B_5$, $^{14}_{7}N_7$

The binding energy B.E of a nucleus is given by

$$BE = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{z(z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \mp E_P$$

The mass of the nucleus is given by

$$M(A,Z) = ZM_H + NM_n - \frac{1}{c^2}\left(a_v A - a_s A^{\frac{2}{3}} - a_c \frac{z(z-1)}{A^{\frac{1}{3}}} - a_n \frac{(A-2Z)^2}{A} \mp E_P \right)$$

The above equation is known as the Semi-empirical mass Formula.

The value of five constants are evaluated from five nuclear masses, the values are

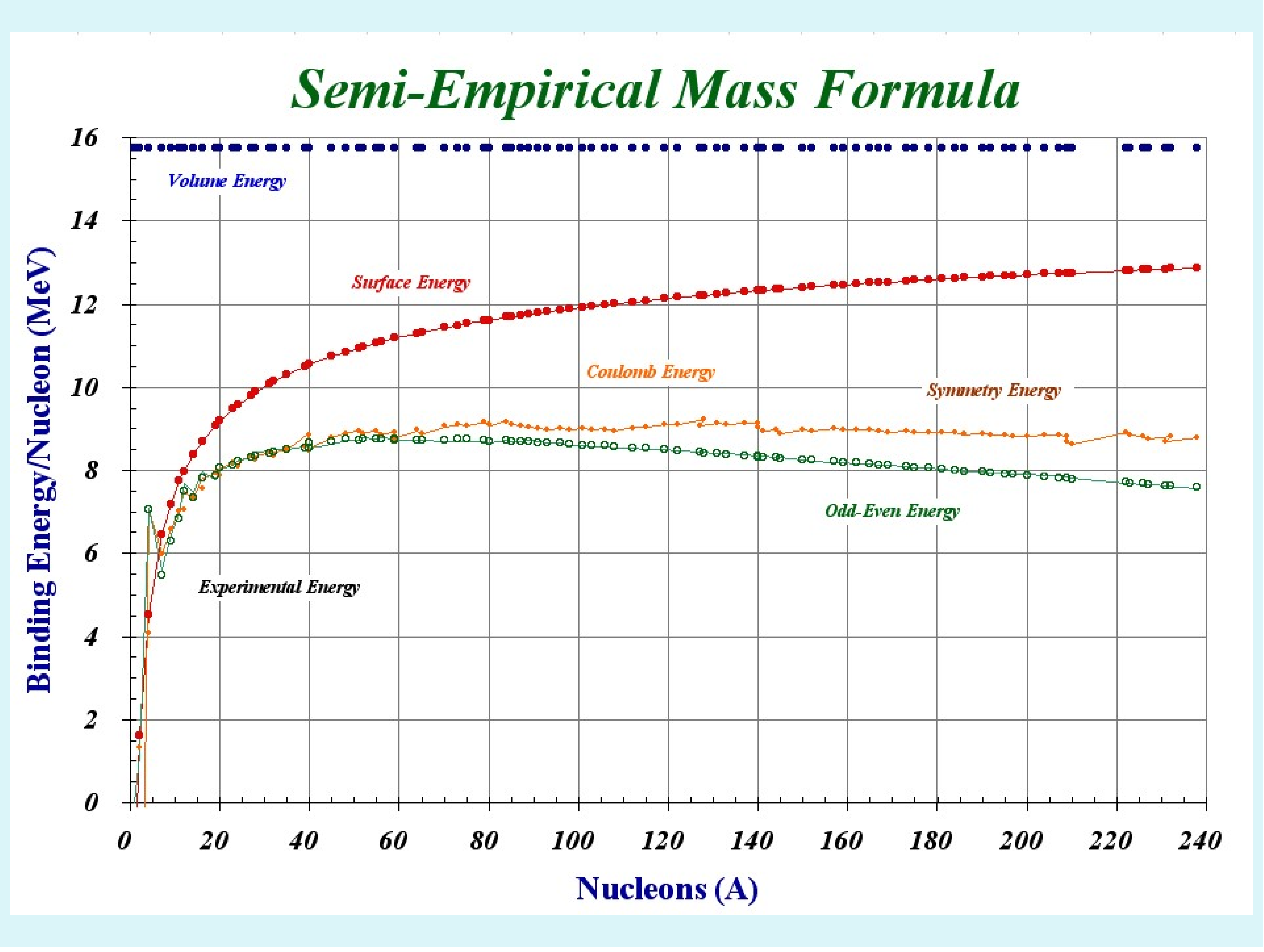
$$a_v = 15.76\ Mev$$

$$a_s = 17.81\ Mev$$

$$a_c = 0.711\ Mev$$

$$a_n = 23.702\ Mev$$

$$a_P = 34\ Mev$$


Semi-Empirical Mass Formula
Volume Energy
Surface Energy
Coulomb Energy
Symmetry Energy
Odd-Even Energy
Experimental Energy
Binding Energy/Nucleon (MeV)
0
2
4
6
8
10
12
14
16
0
20
40
60
80
100
120
140
160
180
200
220
240
Nucleons (A)