VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC accredited 'A' GRADE

Topic: Postulate of Quantum mechanics

Course Title: Elements of Modern Physics

Paper:PHS-A-CC-4-9-TH

Unit: 2

Semester: 4

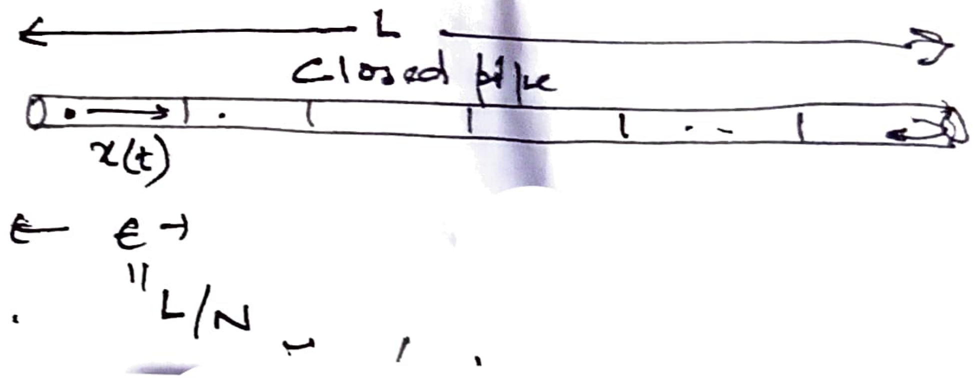
Name of the Teacher:Arvind Pan

Name of the Department:Physics

For particle to be in the 1st int.

$$\vec{e}_1 = \begin{pmatrix}1\\0\\0\\ \vdots\\0\end{pmatrix}$$

N dim vector



For particle to be in the second int.

$$\vec{e}_2 = \begin{pmatrix}0\\1\\0\\ \vdots\\0\end{pmatrix}$$

For particle to be in the nth interval

$$\vec{e}_N = \begin{pmatrix}0\\0\\ \vdots\\1\end{pmatrix}$$

$$\vec{v} = a\vec{e}_1 + b\vec{e}_2 = \begin{pmatrix}a\\b\\0\\ \vdots\\0\end{pmatrix}$$

$$\vec{\psi} = \psi_1\vec{e}_1 + \psi_2\vec{e}_2 + \cdots \psi_N\vec{e}_N$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} \qquad P_n = \psi_n^* \psi_n$$

Norm of a vector $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$

$$|\vec{v}|^2 = (v_1 \; v_2 \; \dots \; v_N) \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix} = v_1^2 + v_2^2 + \dots + v_N^2$$

For $v \to$ complex $\quad \vec{v} \cdot \vec{v} = (v_1^* \; v_2^* \dots v_N^*) \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$

$$= v_1^* v_1 + v_2^* v_2 + \dots v_N^* v_N$$

$$|\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} \qquad \langle\psi| = (\psi_1^* \; \psi_2^* \dots \psi_N^*)$$

$$\langle\psi|\psi\rangle = (\psi_1^* \; \psi_2^* \dots \psi_N^*) \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_N \end{pmatrix}$$

$$= \psi_1^* \psi_1 + \psi_2^* \psi_2 + \dots + \psi_N^* \psi_N$$

$$= P_1 + P_2 + \dots P_N = P_{total} = 1$$

$$\langle\psi|\psi\rangle = \int_0^L \psi^* \psi \, dx = 1$$

$\langle f|g\rangle = \int f^*(x) g(x) \, dx = 0$ if $f(x)$ is orthogonal to $g(x)$

POSTULATE 1: Representation of state : $\psi(x)$

Hermitian Operators: $\hat{A} \to$ operator

$$\int \psi_m^* \hat{A} \psi_n \, dx = \int (\hat{A}^* \psi_m^*) \psi_n \, dx$$

$$\langle\psi_m|\hat{A}|\psi_n\rangle = \langle\hat{A}^\dagger \psi_m|\psi_n\rangle$$

$$\hat{A} = \hat{A}^\dagger$$

$$\hat{A}\psi_n = \underline{a_n}\psi_n \quad ; \quad \hat{A}\psi_m = \underline{a_m}\psi_m$$

POSTULATE 2: PHYSICALLY ~~OBSERBALBLE~~ OBSERVALBLE $\to$ Hermitian Operator

Eigen values of Hermitian operators are real

Proof: $\hat{A}\psi_n = a_n \psi_n$

Multiply by ψ_n^* from left & int. w.r.t x from $-\infty$ to $+\infty$

$$\int_{-\infty}^{+\infty} \psi_n^* \hat{A} \psi_n \, dx = a_n \int_{-\infty}^{+\infty} \psi_n^* \psi_n \, dx = a_n \quad -(1)$$

Using the Hermiticity prop. i.e writing the above integral as

$$\int_{-\infty}^{+\infty} (\hat{A}^* \psi_n^*) \psi_n \, dx = a_n^* \int_{-\infty}^{+\infty} \psi_n^* \psi_n \, dx = a_n^* \quad -(2)$$

For $\hat{A}$ to be Hermitian eqn (1) & (2) must be identical

i.e $a_n = a_n^*$ → Eigenvalues real.

Eigen functions of $\hat{A}$ belonging to distinct eigen values are orthogonal

(1) $\hat{A}\psi_n = a_n \psi_n$ (2) $\hat{A}\psi_m = a_m \psi_m$

Now multiply eqn (1) with ψ_m^* from left & int. w.r.t x from $-\infty$ to $+\infty$

$$\int_{-\infty}^{+\infty} \psi_m^* \hat{A} \psi_n \, dx = a_n \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx \quad -(3)$$

if $\hat{A}$ be Hermitian then eqn (3) gives

$$\int_{-\infty}^{+\infty} (\hat{A}^* \psi_m^*) \psi_n \, dx = a_m^* \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx = a_m \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx \quad -(4)$$

[$a_m^* = a_m$ as $\hat{A}$ is Hermitian]

Again for $\hat{A}$ to be Hermitian eqn(3) = eqn(4). Hence

$$a_m \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx = a_n \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx$$

$$\Rightarrow (a_m - a_n) \int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx = 0 \Rightarrow \text{Since } a_m \neq a_n \text{ so}$$

$$\int_{-\infty}^{+\infty} \psi_m^* \psi_n \, dx = 0$$