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NAAC ACCREDITED 'A' GRADE

Topic: Size and structure of atomic nucleus and its relation with atomic weight

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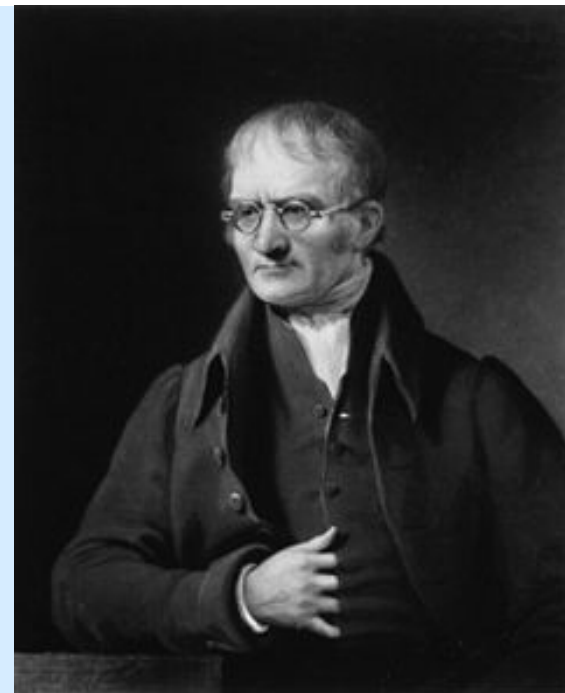
Size and structure of atomic nucleus and its relation with atomic weight

- An Introduction Nuclear Physics
- Size of the Nucleus
- Nuclear density
- Unit of atomic mass
- Nuclear mass
- Nuclear Binding Energy

John Dalton (1766-1844)

In **1803** Proposed his atomic theory

All matter is made of atoms. Atoms are indivisible and indestructible.



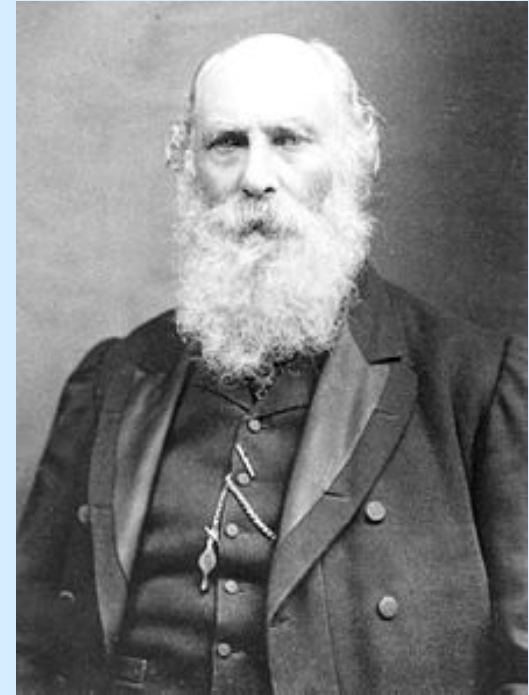
Richard Laming (A British Philosopher)

In **1838** he first introduced the concept of an indivisible quantity of electric charge to explain the chemical properties of atom.

George Johnstone Stoney

An Irish Physicist

In **1891** he named this charge as electron.



J. J. Thomson

In **1897** he and his team identified the particle electron.



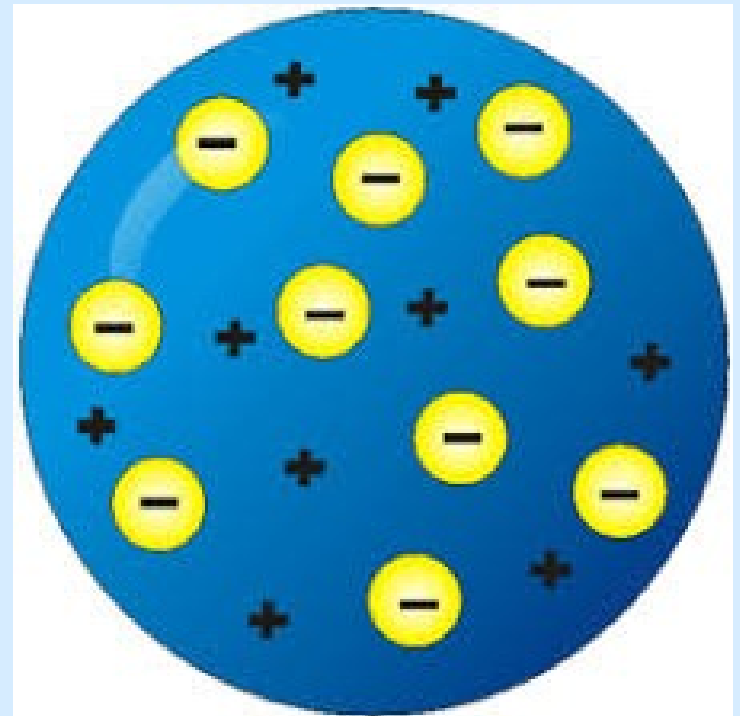
- Atoms have negatively charged particle named electron inside it.

Electron is negative; but atom is neutral

So there must be positive charges inside the atom.

In 1904 Thomson proposed his plum pudding model of atom.

Atom is a sphere of positive charge and some negatively charged electrons are at different position inside it.

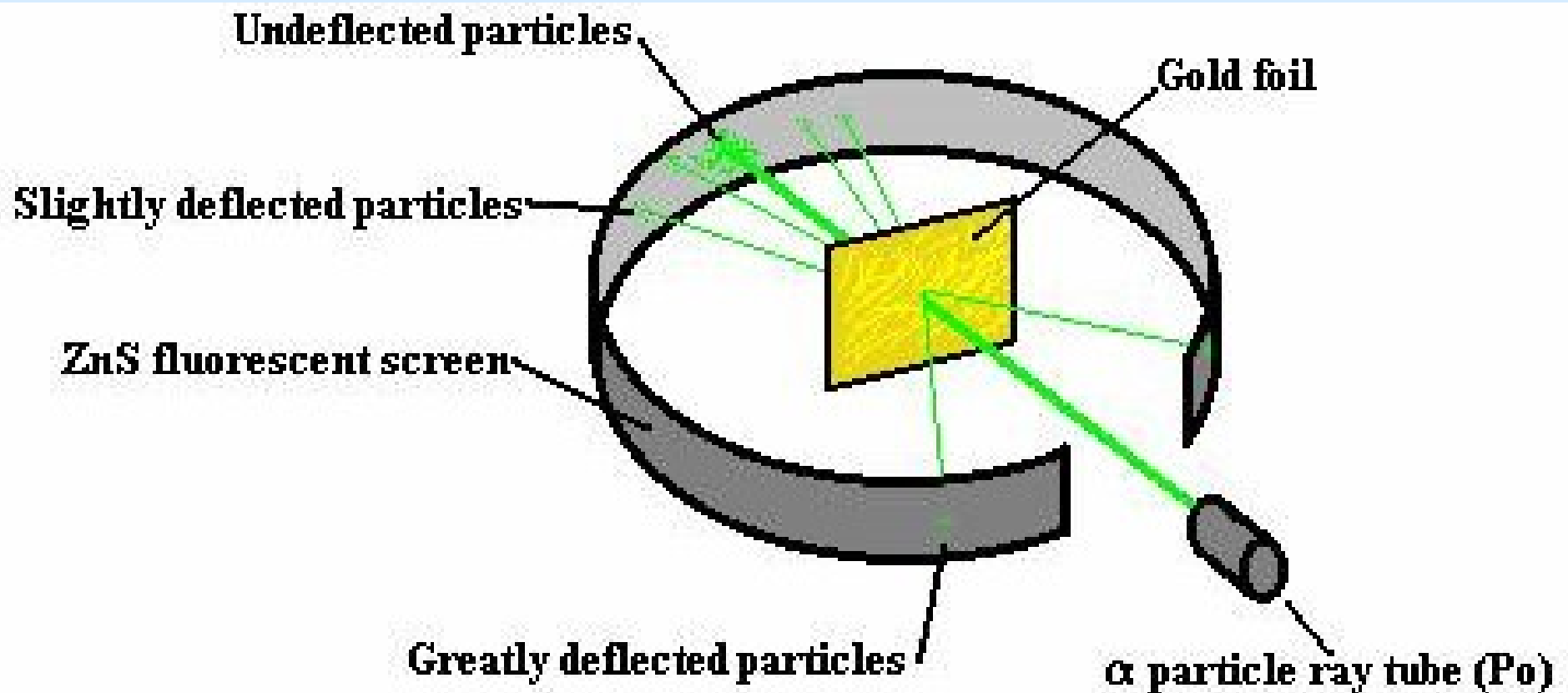


1909- H. Geiger And E Marsden- Gold foil experiment

On a diffuse Reflection of the α - Particle

Proc. Royal Society

Diffuse – sent out in all direction



1. They used different metals to see whether the no. of scattered α particle per minute is co-related with atomic weight.
2. They used Gold Foil of different thickness. To see whether it comes from surface or from interior.
3. Fraction of α - particle back scattered.

1 in 8000 α particle backscattered

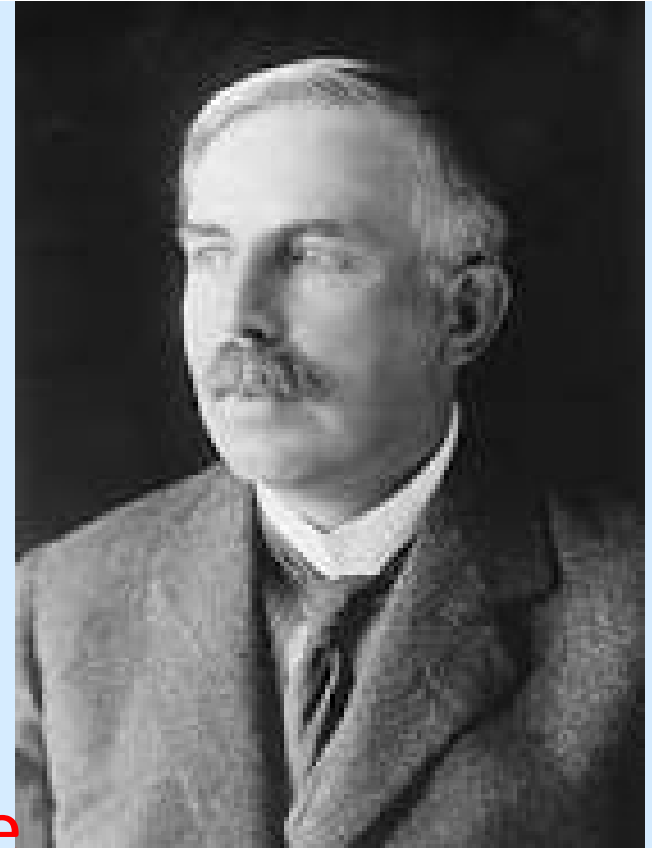
Different metal: (1) Lead (2) Gold
(3) Platinum (4) Tin (5) Silver (6) Copper
(7) Iron (8) Aluminum

1911- Ernest Rutherford

The scattering of α and β particles
by matter and the structure of the
Atom

Philosophical Magazine

There he proposed that atom has
positive charge Ne at the center
and compensating negative charge
is Distributed through the volume.



The Nuclear Physics starts it's journey

He pointed in his paper anything less than 10^{-12} cm is a point.

Size of the nucleus expected by Rutherford is to be 10^{-12} to 10^{-15} m

Size of the Nucleus:

The size of the nucleus was first estimated from Rutherford's α -ray scattering. At the distance of closest approach R the Kinetic energy (K) of α – particle be equal to the repulsive Coulomb energy between α particle and the nucleus.

Therefore at the distance of closest approach R

$$K = \frac{2e Ze}{4\pi\epsilon_0 R} = \frac{2Ze^2}{4\pi\epsilon_0 R}$$

$$R = \frac{2Ze^2}{4\pi\epsilon_0 K}$$

If Z= 79 (Gold) K= 5 Mev

$$\frac{e^2}{4\pi\epsilon_0} = \frac{(1.6 \times 10^{-19} C)^2}{4\pi\epsilon_0}$$

$$= (1.6 \times 10^{-19} C)^2 (9 \times 10^9) Nm^2/C^2$$

$$= (1.6 \times 10^{-19})^2 (9 \times 10^9) Jm$$

$$= \frac{(1.6 \times 10^{-19})^2 (9 \times 10^9)}{1.6 \times 10^{-13}} MeVm$$

$$= 1.6 \times 9 \times 10^{-16} MeVm$$

$$= 1.44 \times 10^{-15} MeVm$$

$$= 1.44 MeVfm$$

$$R = \frac{2 \times 79 \times 1.44 MeVfm}{5 MeV} = 45.5 fm$$

Unit of atomic mass:

The unit of atomic mass is defined to be one-twelfth of the mass of the atom of carbon isotope ^{12}C

The unit of atomic mass is

$$1U = \frac{1}{12} \times \frac{12 \times 10^{-3}}{N_A} = \frac{10^{-3}}{6.02214 \times 10^{23}} \text{Kg}$$
$$= 1.660566 \times 10^{-27} \text{kg}$$

The energy-equivalent of 1u is

$$1U = 1.66 \times 10^{-27} \times c^2$$

$$\begin{aligned}
 1U &= 1.66 \times 10^{-27} \times 8.99 \times 10^{16} \text{kgm}^2 \text{s}^{-2} \\
 &= 14.92 \times 10^{-11} \text{J} \\
 &= \frac{14.92 \times 10^{-11}}{1.60 \times 10^{-13}} \text{Mev} \\
 &= 931.50 \text{ MeV}
 \end{aligned}$$

Constituents of nuclei:

1920- Rutherford - Proton

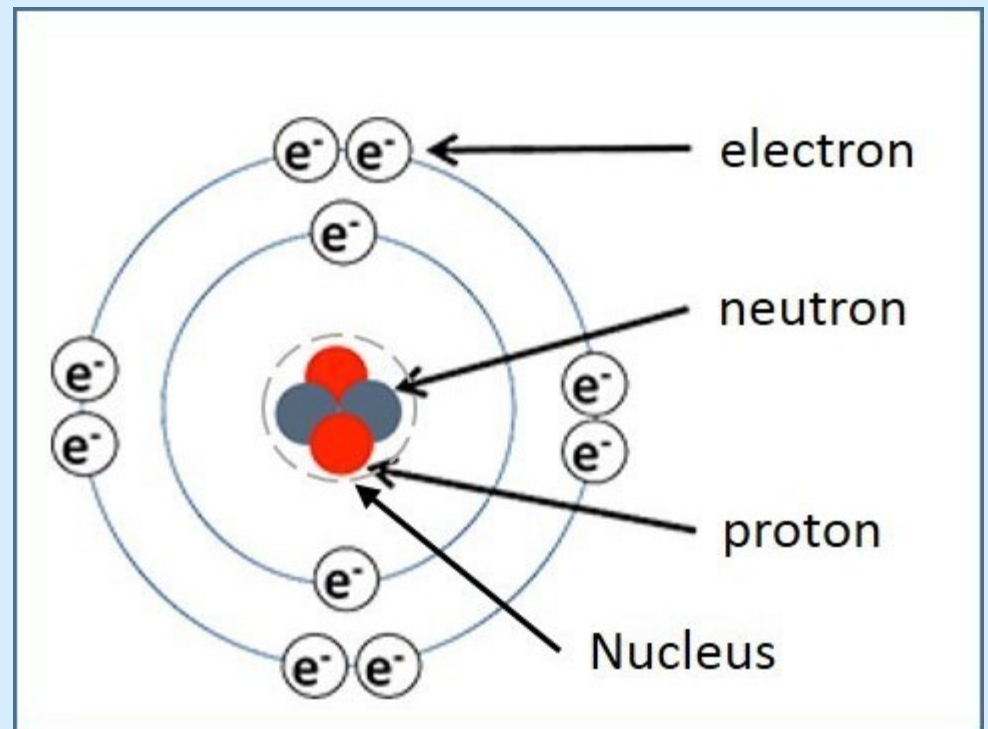
1932- Chadwick- Neutron

Atomic no. Z - Proton No.

Neutron No. N

Mass no. $A = N + Z$

Nucleus - ${}^A_Z X$



The rest mass of electron, proton and neutron are given below:

$$\begin{aligned}\text{Mass of Electron } (m_e) &= 9.11 \times 10^{-31} \text{ Kg} \\ &= 5.48 \times 10^{-4} \text{ U} \\ &= 0.511 \text{ Mev}\end{aligned}$$

$$\begin{aligned}\text{Mass of Proton } (m_p) &= 1.6726 \times 10^{-27} \text{ Kg} \\ &= 1.007 \text{ U} \\ &= 938.27 \text{ Mev}\end{aligned}$$

$$\begin{aligned}\text{Mass of Neutron } (m_n) &= 1.6749 \times 10^{-27} \text{ Kg} \\ &= 1.008 \text{ U} \\ &= 939.57 \text{ Mev}\end{aligned}$$

Nuclear mass:

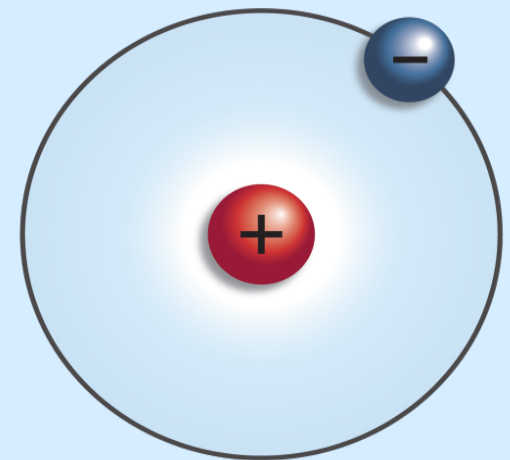
Mass- spectroscopic measurements does not give the masses of nuclei but gives the that of atom. Thus the nuclear mass M_{nuc} is obtained from the atomic mass $M(A,Z)$ by subtracting the masses of z orbital electrons by the following equation

$$M_{\text{nuc}} = M(A,Z) - zm_e + \frac{BE^{\text{at}}}{c^2}$$

$$M_p + m_e - B.E = M_H$$

$$M_{\text{nuc}} + m_e - B.E = M_H$$

$$M_{\text{nuc}} = M_H - m_e + B.E$$



Hydrogen Atom

Nuclear Binding Energy:

Nuclear binding energy is the minimum energy required to split the nucleus of an atom into its component parts i.e. In Z-protons and N- neutrons

Usually the nuclear mass of an atom is less than the sum of the individual masses of the constituent protons and neutrons.

This mass difference is a measure of the nuclear binding energy.

If the mass difference or mass loss be ΔM , then

$$\text{Binding energy} = \Delta M C^2$$

For a ${}^A_Z X$ nucleus

$$\Delta M = ZM_p + NM_n - M_{nuc}$$

$$E_B = \Delta M C^2 = (ZM_p + NM_n - M_{nuc})C^2$$

The mass-spectroscopic measurements does not give the masses of nuclei but those of atoms.

So to represent the expression of E_B and ΔM in terms of atomic mass:

$$E_B = (ZM_p + ZM_e + NM_n - M_{nuc} - ZM_e)C^2$$

$$E_B = (ZM_H C^2 + ZBE_H^{at} + NM_n C^2 - M(A, Z)C^2 - BE^{at})$$

where BE_H^{at} Atomic BE of Hydrogen

BE^{at} Atomic BE of ${}^A_Z X$

$$E_B = (ZM_H + NM_n - M(A, Z))C^2 + ZBE_H^{at} - BE^{at}$$

Neglecting the term $ZBE_H^{at} - BE^{at}$

(small and nearly equal order)

$$E_B = (ZM_H + NM_n - M(A, Z))C^2$$

and $\Delta M = (ZM_H + NM_n - M(A, Z))$

Nuclear size:

Experiments suggests that the atomic nuclei are spherical, or nearly so. So, its volume is proportional to the total number of nucleons i.e. mass number A of it.

$$\frac{4}{3}\pi R^3 \propto A \quad \text{or, } R \propto A^{\frac{1}{3}} \quad \text{or, } R = r_0 A^{\frac{1}{3}}$$

Where R is the radius of the nucleus

r_0 is a constant, called nuclear radius parameter

The value of r_0 ranges from $(1.1- 1.5) \times 10^{-15}$ m

Generally we consider it as 1.2 fm

A	R	A	R
1 (H)	1.2 fm	100 (Mo)	5.56 fm
12(C)	2.7 fm	235(U)	7.4 fm
27(Al)	3.6 fm	280	7.8 fm

Nuclear density:

Nuclear density ρ_N can be defined as

$$\rho_N = \frac{\textit{nuclear mass}}{\textit{nuclear volume}}$$

The nuclear mass M_N is approximately equal to Am_N where A is the mass number and m_N the mass of the Nucleon $\approx 1.67 \times 10^{-27}$ Kg

The nuclear volume,

$$V_N = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi \left(r_0 A^{\frac{1}{3}} \right)^3 = \frac{4}{3} \pi r_0^3 A$$

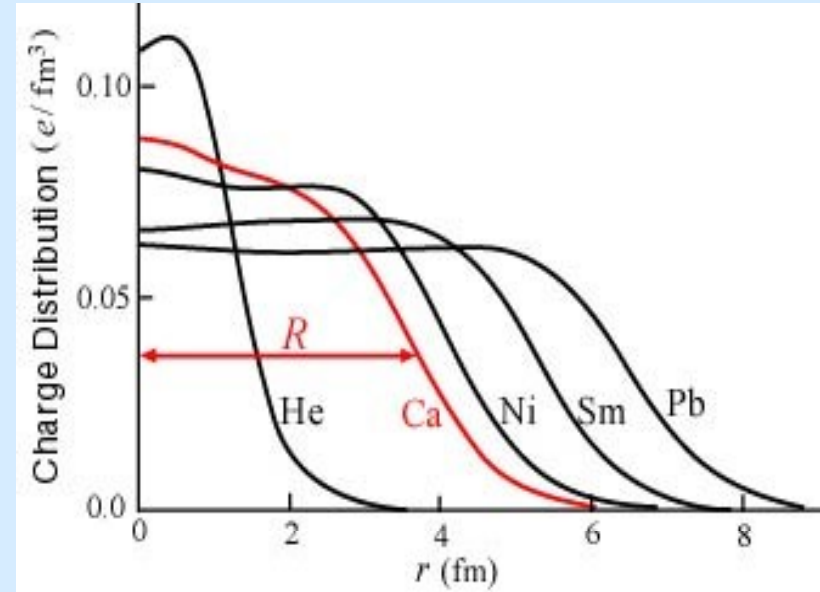
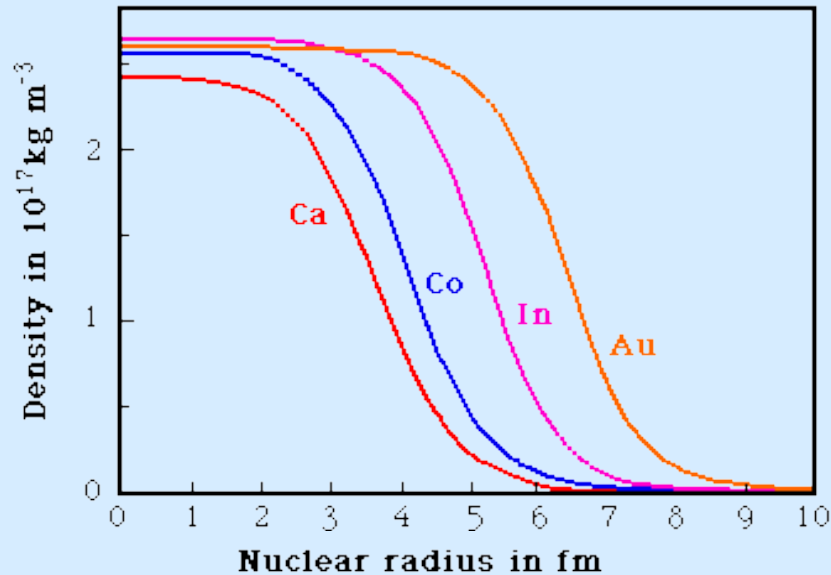
$$\begin{aligned}\rho_N &= \frac{Am_N}{\frac{4}{3}\pi r_0^3 A} = \frac{m_N}{\frac{4}{3}\pi r_0^3} \\ &= \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3} \\ &= 2.30 \times 10^{17} \text{ Kg/m}^3\end{aligned}$$

An unusually large number.

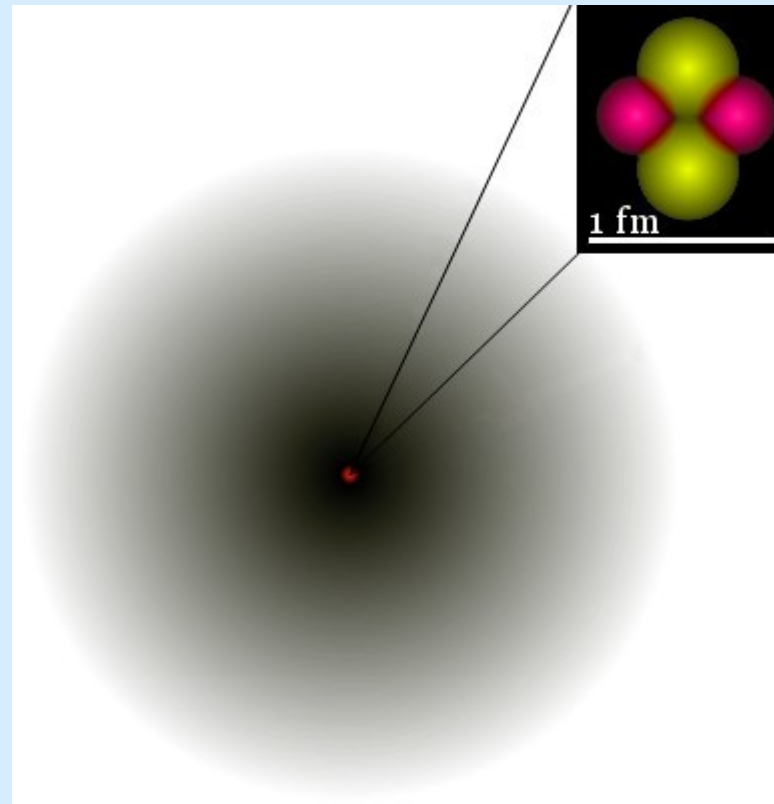
So the nuclear density is extremely high.

As the density of nucleus is independent of A, its value is almost the same for all nuclei.

Distribution of charge and mass in a nucleus



Which indicates that the nucleus does not have the sharp boundary but its boundary is defused



Defused boundary of the nucleus

- **Binding energy of α - particle**

α - particle ie. Helium nucleus is made up of 2 protons and 2 neutrons

$$ZM_p = 2 \times 1.007276 = 2.014552u$$

$$NM_n = 2 \times 1.008665 = 2.017330u$$

$$\text{Total} = 4.031882u$$

$$\text{Atomic mass of } {}^4_2\text{He} = 4.002603u$$

$$\text{Difference} = 0.029279u$$

Since $1u \approx 931 \text{ Mev}$

$$E_{B\alpha} = 0.029279 \times 931 \text{ Mev} = 27.16 \text{ Mev}$$

Which explains why the He-nucleus is very stable.