VIVEKANANDA COLLEGE
THAKURPUKUR
KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

Topic: Introduction of plasma physics

Course Title: Introductory Plasma Physics

Paper: Classical Electrodynamics

Unit: PHY 421

Semester: Second (M.Sc.)

Name of the Teacher: Laxmikanta Karmakar

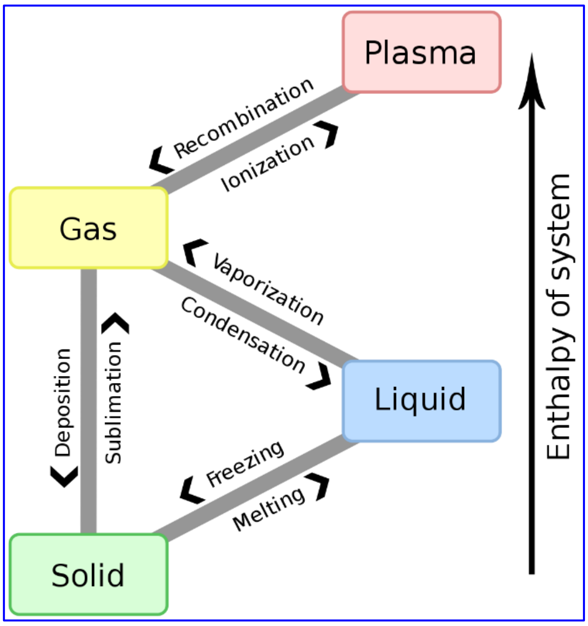
Name of the Department: Physics

Introductory Plasma Physics

Definition of plasma; Its occurrence in nature; Dilute and dense plasma; Uniform but time-dependent magnetic field: Magnetic pumping; Static non-uniform magnetic field: MHD equations, Pinched plasma; Plasma Oscillations. *[6 lecture hours]*

Reference Books: ***(i) J.A. Bittencourt: Fundamentals of Plasma Physics***
(ii) F.F. Chen: Introduction to Plasma Physics and Controlled Fusion
(iii) J.D. Callen: Fundamentals of Plasma Physics
(iv) S.N. Sen: Plasma Physics

At first see the schematic diagram of phase change of any matter in different states (solid → liquid → gas → plasma). We know that, during any phase transformations between solid-liquid-gas phases, the temperature of the mater must constant, for example when ice (solid) melt into water (liquid) the temperature of system must constant at 0 °C. But in case of ionization of gas (produce plasma) the temperature of system does not constant during the transformation from gas to plasma. That's why it is **difficult to say** that the '*plasma is a fourth state of matter*'.



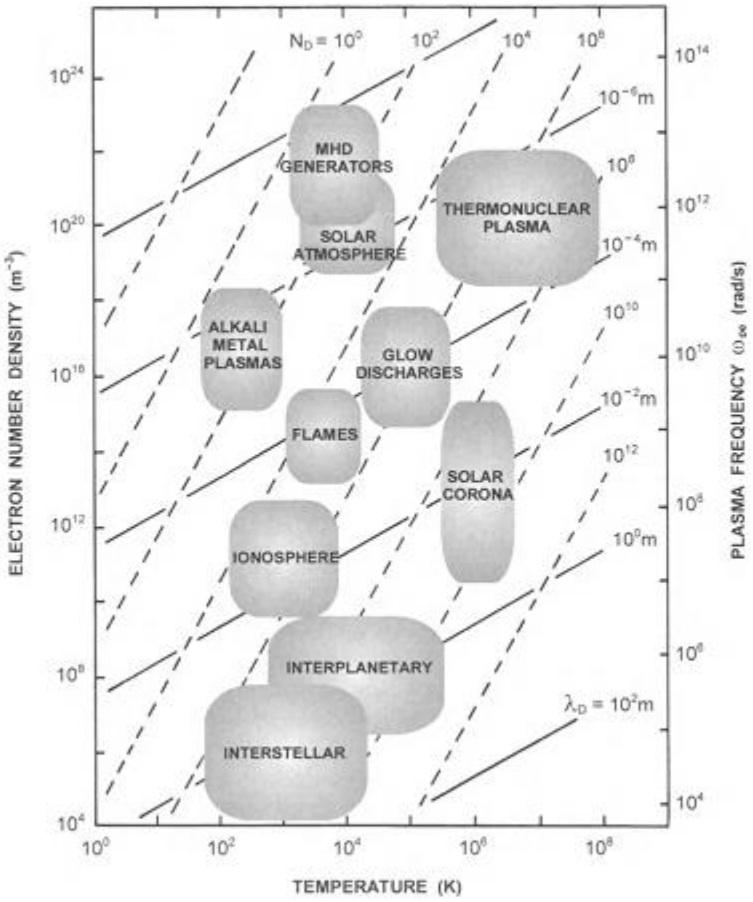
The word '***plasma***' comes from the Greek and means ***something molded***. It was applied for the first time by Tonks and Langmuir, in 1929, to describe the inner region, remote from the boundaries, of a glowing ionized gas produced by electric discharge in a tube, the ionized gas as a whole remaining electrically neutral.

A plasma *(sometime called fourth state of matter)* ***is a quasi-neutral*** *(macroscopically neutral, i.e.; net charge is zero)* ***gas of charged particles*** *(ions and electrons)* ***and neutral atoms or molecules which exhibits collective behaviour***.

❖ Occurrence of Plasma in Nature:

In *astrophysics* and in *theoretical* physics during the last century, it was realized that most of the matter in the known universe, with a few exceptions such as the surface of cold planets exists as a plasma.

The **sun**, is a plasma phenomenon. Its energy output is derived from thermonuclear fusion reactions of protons forming helium ions deep in its interior, where temperatures > 1.2 x 10^7 K. The high temperature of its interior and the consequent thermonuclear reactions keep the entire sun gaseous. Due to its large mass (2 x 10^{30} kg), the sun's gravitational force is sufficient to prevent the escape of all but the most energetic particles and, of course, radiation from the hot solar plasma. There is no sharp boundary surface to the sun. Its visible part is known as the *solar atmosphere*, which is divided into three general regions or layers. The photosphere, with a temperature of about 6,000 K, comprises the visible disk, the layer in which the gases become opaque, and is a few hundred kilometres thick. Surrounding the photosphere there is a reddish ring called the chromosphere, approximately 10,000 km thick, above which flame-like prominences rise with temperatures of the order of 100,000 K. Surrounding the chromosphere there is a tenuous hot plasma, extending millions of kilometres into space, known as the corona. A steep temperature gradient extends from the chromosphere to the hotter corona, where the temperature exceeds 10^6 K. The sun possesses a variable magnetic field, which at its surface is typically of the order of 10^{-4} tesla, but in the regions of sunspots (regions of relatively cooler gases) the solar magnetic field rises to about 0.1 tesla. The others natural plasma occurs during (i) lighting, (ii) Arora Borealis, (iii) earth ionosphere etc.



Ranges of temperature and electron density for several laboratory and cosmic plasmas and their characteristic physical parameters: Debye length λ_D, plasma frequency ω_{pe}, and number of electrons N_D in a Debye sphere. MHD, magnetohydrodynamic.

❖ Plasma Production:

A plasma can produced by raising the temperature until reasonable high fractional ionization. Under thermodynamic equilibrium the degree of ionization and electron temperature are closely related as known as 'Saha equation':

$$\frac{n_i}{n_n} = 2.405 \times 10^{21} T^{3/2} \frac{1}{n_i} exp\left(-\frac{U}{kT}\right)$$

$$\Rightarrow n_i^2 = 2.405 \times 10^{21} T^{3/2} n_n exp\left(-\frac{U}{kT}\right) \qquad (1)$$

The plasma can create in the laboratory. Depending on production method, plasma can high or low density, high or low temperature, stable or unstable. **Dense plasma** behaves like fluid and treated as a whole property. In case of **Dilute plasma** considering the property of individual particles.

The common process to produce artificial plasma are (i) ***Photoionization of gas***, and (ii) ***Electric discharge in gas***. If the energy of incident photon is greater or equal to the ionization energy of gas, then the gas can ionize and the excess energy of incident photon converted into the kinetic energy of generated electron-ion pair. This photoionization of gas type plasma observed in the ionosphere of the Earth's atmosphere. For electric discharge in gas, apply external electric field within gas and the electrons accelerated and radiated also collide with ions and neutral particles. Since the mass of electron is lesser than the mass of ion, so the kinetic energy of electron is greater than that of ion and the corresponding temperature of electron also higher than that of ions. In general, the laboratory produce plasma by the electric discharge method.

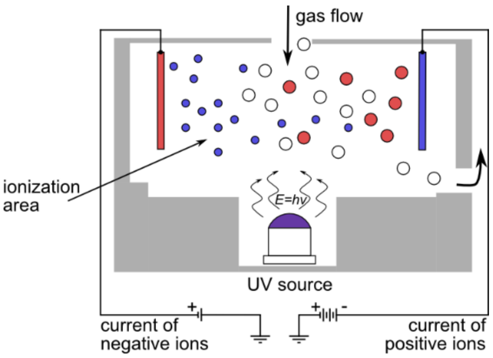
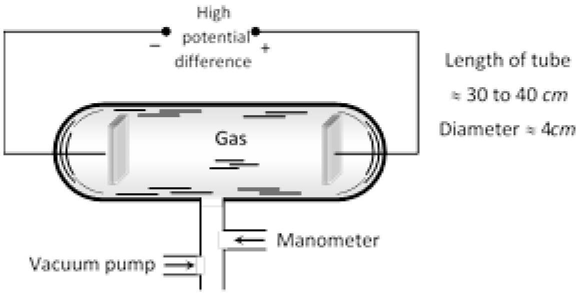


Photo ionization of gas



Electric discharge in gas

❖ Particle interaction and collective effect:

Within the plasma there are two charge-charge and one charge-neutral interaction happens. The charge-charge interactions are, **(i) Coulomb interaction**: the heavy ions produce electric field and prepared the electrostatic forces on charged electrons and others ions, and **(ii) Lorentz force**: since the electrons (also ions) are moving within plasma, produce current and produce corresponding magnetic field. This magnetic field produce Lorentz force on charged particle. And the charge-neutral interaction is **(iii) polarization**: the charge particles polarized the neutral part of plasma.

Depending on particle interaction plasma classified into two categories: (i) **Weakly ionized**: number of charge-neutral interaction> number of charge-charge interaction. And, (ii) **Strongly ionized**: number of charge-neutral interaction< number of charge-charge interaction.

❖ Basic plasma phenomena:

(i) Plasma is good electrical conductor (since the electron mobility within plasma is very high) and also a good thermal conductor.

(ii) Due to lower mass of electron than ion, the diffusion rate of electrons is higher than that of ion and making charge separation and produce polarized electric field. This additional electric field enhance the diffusion rate of ions, and as a whole the diffusion rate of electrons and ions are nearly same, this type diffusion known as '***Ambipolar diffusion***'. This diffusion can be reduced by applying external strong magnetic field and finally the plasma can confine within a region. The diffusion coefficient is inversely proportion to $\boldsymbol{B}^2$. But, in case of ***Bohm diffusion***, the diffusion coefficient is inversely proportion to $\boldsymbol{B}$.

(iii) Due to collision between charge-charge and charge-neutral particle, the amplitude of electromagnetic (EM) wave is reduced (damping). During the collision the energy of EM wave transfer to the plasma particle. These phenomena known as '***Landau damping***'. The opposite phenomena also happen when the energy transfer from plasma particle to the EM wave. For that occurrence, the plasma become ***instability***.

❖ Radiation from plasma:

(i) **Radiation from emitting atoms and molecules**: The black body radiation emitted from plasma in thermodynamic equilibrium. Ionization and recombination both occur. So, radiation comes when recombination happens. This radiation constitutes the line spectra of plasma.

(ii) **Radiation from accelerated charge particles**: when charged particles collide with neutral atoms then occur '***Bremsstrahlung***'. This type radiation happening in three ways:

(1) Charge particle unbound before and after collision, is called as, ***free-free bremsstrahlung***.

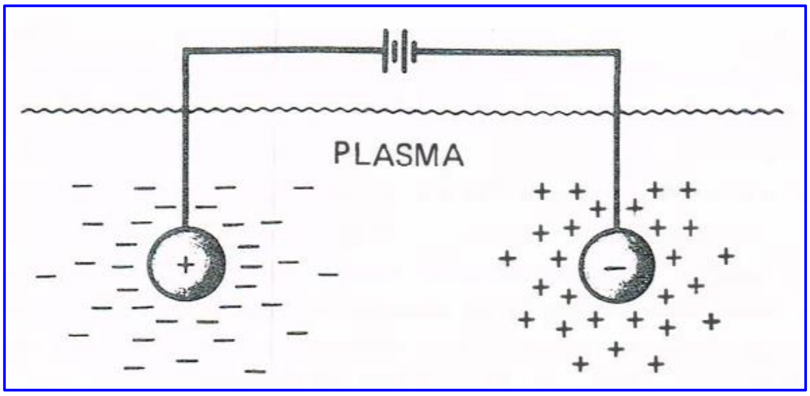
(2) If free charge particle bound after collision, then this is called as, ***free-bound radiation***.

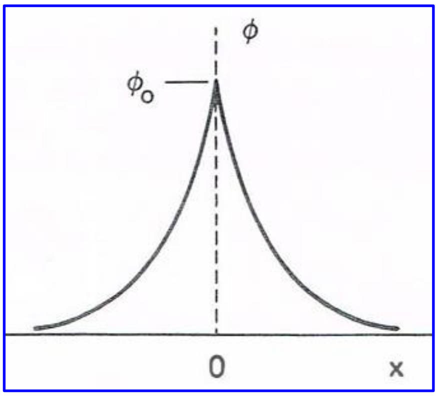
(3) In magnetic field, charge particle rotated in spiral path, called ***Cyclotron radiation***.

Criteria for the definition of a PLASMA

Debye Shielding:

A fundamental characteristic of the behaviour of a plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery (see figure). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround the positive ball. (*We assume that a layer of dielectric keeps the plasma from actually recombining on the surface, or that the battery is large enough to maintain the potential in spite of this.*) If the plasma were cold and there were no thermal motions, there would be just as many charges in the cloud as in the ball; the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temperature is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well. The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy $\boldsymbol{KT}$ of the particles, and the shielding is not complete. Potentials of the order of $\boldsymbol{KT/e}$ can leak into the plasma and cause finite electric fields to exist there.





Let us compute the approximate thickness of such a charge cloud. Imagine that the potential $\boldsymbol{\phi}$ on the plane $\boldsymbol{x} = \boldsymbol{0}$ is held at a value $\boldsymbol{\phi_0}$ by a perfectly transparent grid (see figure). We wish to compute $\boldsymbol{\phi(x)}$. For simplicity, we assume that the ion-electron mass ratio $\boldsymbol{M/m}$ is infinite, so that the ions do not move but form a uniform background of positive charge. To be more precise, we can say that $\boldsymbol{M/m}$ is large enough that, the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} = -\frac{e(n_i-n_e)}{\epsilon_0} \qquad \text{(take, Z = 1)} \qquad (2)$$

If the density far away is n_∞, we have $n_i = n_\infty$.

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A\exp\left[-\frac{\frac{1}{2}mu^2+q\phi}{k_BT}\right]$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating $\boldsymbol{f}(\boldsymbol{u})$ over $\boldsymbol{u}$, setting $\boldsymbol{q} = -\boldsymbol{e}$, and noting that $\boldsymbol{n_e}(\boldsymbol{\phi} \to \mathbf{0}) = \boldsymbol{n}_\infty$, we find

$$\boldsymbol{n_e} = \boldsymbol{n}_\infty \exp\left[\frac{\boldsymbol{e\phi}}{\boldsymbol{k_B T_e}}\right] \tag{3}$$

Substituting for $\boldsymbol{n_i}$ and $\boldsymbol{n_e}$, in eq. (1), we have

$$\nabla^2\boldsymbol{\phi} = -\frac{\rho}{\epsilon_0} = \frac{e n_\infty}{\epsilon_0}\left[\exp\left(\frac{e\phi}{k_B T_e}\right) - 1\right] \tag{4}$$

In the region where $\left|\frac{e\phi}{k_B T_e}\right| \ll 1$, we can expand the exponential in a Taylor series:

$$\nabla^2\boldsymbol{\phi} = \frac{e n_\infty}{\epsilon_0}\left[\left(\frac{e\phi}{k_B T_e}\right) + \frac{1}{2}\left(\frac{e\phi}{k_B T_e}\right)^2 + \cdots\right] \tag{5}$$

No simplification is possible for the region near the grid, where $\left|\frac{e\phi}{k_B T_e}\right|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in eq. (5), we have

$$\nabla^2\boldsymbol{\phi} = \frac{e n_\infty}{\epsilon_0}\frac{e\phi}{k_B T_e} = \frac{e^2 n_\infty}{\epsilon_0 k_B T_e}\boldsymbol{\phi} \tag{6}$$

We can write the solution of above equation as,

$$\boldsymbol{\phi} = \boldsymbol{\phi_0}\exp\left(-\frac{|x|}{\lambda_D}\right) \tag{7}$$

where, $\boldsymbol{\lambda_D} = \left(\frac{\epsilon_0 k_B T_e}{n_\infty e^2}\right)^{1/2}$, called the Debye length, is a measure of the shielding distance or thickness of the sheath.

Note that as the density is increased, $\boldsymbol{\lambda_D}$ decreases, as one would expect, since each layer of plasma contains more electrons. Furthermore, $\boldsymbol{\lambda_D}$ increases with increasing $\boldsymbol{k_B T_e}$. Without thermal agitation, the charge cloud would collapse to an infinitely thin layer. Finally, it is the electron temperature which is used in the definition of $\boldsymbol{\lambda_D}$ because the electrons, being more mobile than the ions, generally do the shielding by moving so as to create a surplus or deficit of negative charge.

If the dimensions $\boldsymbol{L}$ of a system are much larger than $\boldsymbol{\lambda_D}$, then whenever local concentrations of charge arise or external potentials are introduced into the system, these are shielded out in a distance short compared with **L**, leaving the bulk of the plasma free of large electric potentials or fields. Outside of the sheath on the wall or on an obstacle, $\nabla^2\boldsymbol{\phi}$ is very small, and $\boldsymbol{n_i}$ is equal to $\boldsymbol{n_e}$, typically, to better than one part in $\mathbf{10^6}$. It takes only a small charge imbalance to give rise to potentials of the order of $\boldsymbol{k_B T_e}$. The plasma is "***quasi-neutral***"; that is, neutral enough so that one can take $\boldsymbol{n_i} \simeq \boldsymbol{n_e} \simeq \boldsymbol{n}$, where $\boldsymbol{n}$ is a common density called the ***plasma density***, but not so neutral that all the interesting electromagnetic forces vanish.

A criterion for an ionized gas to be a plasma is that it be dense enough that $\boldsymbol{\lambda_D}$ is much smaller than $\boldsymbol{L}$. The phenomenon of **Debye shielding** also occurs-in modified form-in single-species systems, such as the electron streams in klystrons and magnetrons or the proton beam in a cyclotron. In such cases, any local bunching of particles causes a large unshielded electric field unless the density is extremely low (which it often is). An externally imposed potential-from a wire probe, for instance-would be shielded out by an adjustment of the density near the electrode. Single-species systems, or un-neutralized plasmas, are not strictly plasmas; but the mathematical tools of plasma physics can be used to study such systems.

It is convenient to define a ***Debye sphere*** as a sphere inside the plasma of radius equal to $\boldsymbol{\lambda_D}$. Any electrostatic fields originated outside a Debye sphere are effectively screened by the charged particles and do not contribute significantly to the electric field existing at its centre. Consequently, each charge in the plasma interacts collectively only with the charges that lie inside its Debye sphere, its effect on the other charges being effectively negligible. The number of electrons $\boldsymbol{N_D}$, inside a Debye sphere, is given by

$$\boldsymbol{N_D = \frac{4}{3}\pi\lambda_D^3 n_e = \frac{4}{3}\pi\left(\frac{\epsilon_0 k_B T_e}{n_e^{1/3} e^2}\right)^{3/2}} \tag{8}$$

The Debye shielding effect is a characteristic of all plasmas, although it does not occur in every medium that contains charged particles. A necessary and obvious requirement for the existence of a plasma is that the physical dimensions of the system be large compared to $\boldsymbol{\lambda_D}$. Otherwise there is just not sufficient space for the collective shielding effect to take place, and the collection of charged particles will not exhibit plasma behaviour. If $\boldsymbol{L}$ is a characteristic dimension of the plasma, a ***first criterion*** for the definition of a plasma is therefore, $\boldsymbol{L \gg \lambda_D}$.

Since the shielding effect is the result of the collective particle behaviour inside a Debye sphere, it is also necessary that the number of electrons inside a Debye sphere be very large. A ***second criterion*** for the definition of a plasma is therefore, $\boldsymbol{n_e\lambda_D^3 \gg 1}$.

This means that the average distance between electrons, which is roughly given by $\boldsymbol{n_e^{-1/3}}$, must be very small compared to $\boldsymbol{\lambda_D}$. The quantity defined by $\boldsymbol{g = \frac{1}{n_e\lambda_D^3}}$ is known as the plasma parameter and the condition $\boldsymbol{g \ll 1}$ is called the plasma approximation. This parameter is also a measure of the ratio of the mean interparticle potential energy to the mean plasma kinetic energy.

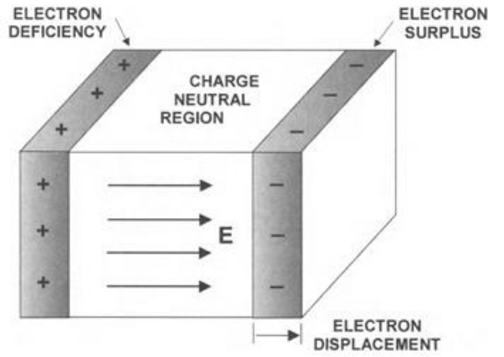
Note that the requirement $(\boldsymbol{L \gg \lambda_D})$ already implies in macroscopic charge neutrality if it is realized that deviations from neutrality can naturally occur only over distances of the order of $\boldsymbol{\lambda_D}$. Nevertheless, macroscopic neutrality is sometimes considered as a ***third criterion*** for the existence of a plasma, although it is not an independent one, and can be expressed as $n_e = \sum_i n_i$.

❖ Plasma Frequency:

An important plasma property is the stability of its macroscopic space charge neutrality. When a plasma is instantaneously disturbed from the equilibrium condition, the resulting internal space charge fields give rise to collective particle motions that tend to restore the original charge neutrality. These collective motions are characterized by a natural frequency of oscillation known as the ***plasma frequency***. Since these collective oscillations are high-frequency oscillations, the ions, because of their heavy mass, are to a certain extent unable to follow the motion of the electrons. The electrons oscillate collectively about the heavy ions, the necessary collective restoring force being provided by the ion-electron coulomb attraction. The period of this natural oscillation constitutes a meaningful time scale against which can be compared the dissipative mechanisms tending to destroy the collective electron motions.

Consider a plasma initially uniform and at rest, and suppose that by some external means a small charge separation is produced inside it (see figure 1). When the external disturbing force is removed instantaneously, the internal electric field resulting from charge separation collectively accelerates the electrons in an attempt to restore the charge neutrality. However, because of their inertia, the electrons move beyond the equilibrium position, and an electric field is produced in the opposite direction. This sequence of movements repeats itself periodically, with a continuous transformation of kinetic energy into potential energy, and vice versa, resulting in fast collective oscillations of the electrons about the more massive ions. On the average the plasma maintains its macroscopic charge neutrality. The angular frequency of this collective electron oscillations, called the (electron) plasma frequency, is given by

$$\omega_{pe} = \left(\frac{n_e e^2}{m_e \epsilon_0}\right)^{1/2} \qquad (9)$$



The electric field resulting from charge separation provides the force that generates the electron plasma oscillations.

Collisions between electrons and neutral particles tend to damp these collective oscillations and gradually diminish their amplitude. If the oscillations are to be only slightly damped, it is necessary that the electron neutral collision frequency (ν_{en}) be smaller than the electron plasma frequency, $\nu_{pe} > \nu_{en}$, where $\nu_{pe} = \omega_{pe}/2\pi$. Otherwise, the electrons will not be able to behave in an independent way, but will be forced by collisions to be in complete equilibrium with the neutrals, and the medium can be treated as a neutral gas. The ***fourth criterion*** for the existence of a plasma. This criterion can be alternatively written as $\omega\tau > 1$, where $\tau = 1/\nu_{en}$ represents the average time an electron travels between collisions with neutrals, and ω stands for the angular frequency of typical plasma oscillations. It implies that the average time between electron-neutral collisions must be large compared to the characteristic time during which the plasma physical parameters are changing.