# VIVEKANANDA COLLEGE THAKURPUKUR KOLKATA-700063

NAAC ACCREDITED ‘A’ GRADE

**Topic:** MHD equations and instability of plasma

**Course Title:** Introductory Plasma Physics

**Paper:** Classical Electrodynamics

**Unit:** PHY 421

**Semester:** Second (M.Sc.)

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**Name of the Department:** Physics

# Introductory Plasma Physics

## ❖ MHD equations:

The previous discussions of the trajectory of charge particle of plasma within different electric and magnetic field. This was based on the kinematics of individual charge particle which is used in case of ***dilute plasma***. Now we interest the mechanism of dense plasma which can be treated as the flow of the charge fluid. If such a charge fluid moves in a magnetic field, electric current is induced in the fluid as a result of its motion. The flow of charged fluid across the magnetic field will produce mechanical forces which will modify the motion of the fluid. In this way a coupling occurs between hydrodynamic motion and electromagnetic phenomena, resulting ***magnetohydrodynamics*** **(MHD)** for describing motion of dense plasma under electric and magnetic field.

The motion of the normal fluid is described by the familiar equations of hydrodynamics which represent the conservation law of mechanics. The conservation of matter is described by the law of continuity. If $\rho$ is the density and $v$ the velocity

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}.(\rho\vec{v})=0\Rightarrow\frac{\partial\rho}{\partial t}+\rho\vec{\nabla}.\vec{v}+\vec{v}.\vec{\nabla}\rho=0 \quad (42)$$

The above equation of hydrodynamics relates to gain of momentum in a fluid element to the forces which act upon it. Hence the net force, $$\rho\left(\frac{d\vec{v}}{dt}\right)=-\vec{\nabla}p+\rho\vec{g}+\vec{J}\times\vec{B} \quad (43)$$

where $p$ is the pressure, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic field and the gravitational force is represented by $\rho\vec{g}$, $\frac{d}{dt}$ is the mobile operator so that, $\frac{d}{dt}=\frac{\partial}{\partial t}+\vec{v}.\vec{\nabla}$.

However, when consider the electromagnetic effect, we have to consider the magnetic stress $\left(\frac{J}{c}\times B\right)$. This utilize the generalised version of Ohm's law in which the electric field is taken as that seen an observer moving with the field. Hence total force on charge fluid is sum of the forces due to electric field and Lorentz field. For unit charge the total electric field will be, $\vec{E}^*=\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}$ (44)

Then, $$\vec{\imath}=\sigma\left[\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right] \quad (45)$$

If the fluid is a perfect conductor, $\frac{J}{\sigma}=0$. Hence, $$\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}=0 \quad (46)$$

If we neglect the gravitational and viscous forces in equation [43], but include the force due to electromagnetic effect, $$\rho\left(\frac{d\vec{v}}{dt}\right)=-\vec{\nabla}p+\frac{1}{c}(\vec{J}\times\vec{B})\Rightarrow\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\vec{\nabla})\vec{v}\right)=-\vec{\nabla}p+\frac{1}{c}(\vec{J}\times\vec{B}) \quad (47)$$

Further the Maxwell's equations:

$$\frac{1}{\mu}\vec{\nabla}\times\vec{B}=\frac{4\pi\vec{J}}{c} \quad (48) \qquad \vec{\nabla}\times\vec{E}=-\frac{1}{c}.\frac{\partial\vec{B}}{\partial t} \quad (49)$$

$$\vec{\nabla}.\vec{B}=0 \quad (50) \qquad \vec{\nabla}.\vec{E}=\frac{4\pi e}{\varepsilon} \quad (51)$$

From equation (42), $\rho$ if is invariant, $$\rho\vec{\nabla}.\vec{v}=0\Rightarrow\vec{\nabla}.\vec{v}=0 \quad (52)$$

Since we are dealing with a conducting fluid moving in a magnetic field, we can transform the equations in terms of the parameters, $\rho$, $v$, $p$ and $B$ and eliminate $E$ and $\boldsymbol{i}$ from equation (48); $\vec{J}=\frac{c}{\mu}\frac{1}{4\pi}\vec{\nabla}\times\vec{B}$.

Putting the value of $\boldsymbol{i}$ in equation (47); $$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\vec{\nabla})\vec{v}\right)=-\vec{\nabla}p+\frac{1}{c}\left(\frac{c}{\mu}\frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}\right)$$

$$= -\vec{\nabla}p + \frac{1}{4\pi\mu}\left[\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B}\right] = -\vec{\nabla}p + \frac{1}{4\pi\mu}\vec{B}\left(\vec{\nabla}.\vec{B}\right) - \frac{1}{4\pi\mu}\vec{\nabla}\left(\vec{B}.\vec{B}\right) \qquad (53)$$

Now from equation (48), $\frac{1}{\mu}\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \frac{4\pi}{c}\vec{\nabla}\times\vec{J} = \frac{4\pi}{c}\vec{\nabla}\times\left[\sigma\left\{\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right\}\right]$

$$\Rightarrow \frac{1}{\mu}\left[\vec{\nabla}\left(\vec{\nabla}.\vec{B}\right) - \nabla^2\vec{B}\right] = \frac{4\pi\sigma}{c}\left[\vec{\nabla}\times\vec{E} + \vec{\nabla}\times\left(\frac{1}{c}\vec{v}\times\vec{B}\right)\right]$$

$$\Rightarrow -\frac{1}{\mu}\nabla^2\vec{B} = \frac{4\pi\sigma}{c^2}\left[c\left(\vec{\nabla}\times\vec{E}\right) + \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\right] \qquad [\textit{using eq. (50)}]$$

$$\Rightarrow \frac{1}{\mu}\nabla^2\vec{B} = \frac{4\pi\sigma}{c^2}\left[\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\right] \qquad [\textit{using eq. (49)}]$$

$$\Rightarrow \frac{1}{\mu}\nabla^2\vec{B} = \frac{4\pi\sigma}{c^2}\left[\frac{\partial\vec{B}}{\partial t} - \vec{v}\left(\vec{\nabla}.\vec{B}\right) + \vec{B}\left(\vec{\nabla}.\vec{v}\right) - \left(\vec{B}.\vec{\nabla}\right)\vec{v} + \left(\vec{v}.\vec{\nabla}\right)\vec{B}\right] \quad [\textit{using vector identity}]$$

$$\Rightarrow \frac{1}{\mu}\nabla^2\vec{B} = \frac{4\pi\sigma}{c^2}\left[\frac{\partial\vec{B}}{\partial t} + \vec{B}\left(\vec{\nabla}.\vec{v}\right) - \left(\vec{B}.\vec{\nabla}\right)\vec{v} + \left(\vec{v}.\vec{\nabla}\right)\vec{B}\right] \qquad [\textit{using eq. (50)}]$$

$$\Rightarrow \frac{c^2}{4\pi\sigma\mu}\nabla^2\vec{B} = \frac{\partial\vec{B}}{\partial t} + \vec{B}\left(\vec{\nabla}.\vec{v}\right) - \left(\vec{B}.\vec{\nabla}\right)\vec{v} + \left(\vec{v}.\vec{\nabla}\right)\vec{B} \qquad (54)$$

Thus, we have the modified MHD equations as follows:

$$\frac{\partial\rho}{\partial t} + \rho\vec{\nabla}.\vec{v} + \vec{v}.\vec{\nabla}\rho = 0 \qquad \textbf{(42)}$$

$$\rho\left(\frac{\partial\vec{v}}{\partial t} + \left(\vec{v}.\vec{\nabla}\right)\vec{v}\right) = -\vec{\nabla}p + \frac{1}{4\pi\mu}\vec{B}\left(\vec{\nabla}.\vec{B}\right) - \frac{1}{4\pi\mu}\vec{\nabla}\left(\vec{B}.\vec{B}\right) \qquad \textbf{(53)}$$

$$\frac{c^2}{4\pi\sigma\mu}\nabla^2\vec{B} = \frac{\partial\vec{B}}{\partial t} + \vec{B}\left(\vec{\nabla}.\vec{v}\right) - \left(\vec{B}.\vec{\nabla}\right)\vec{v} + \left(\vec{v}.\vec{\nabla}\right)\vec{B} \qquad \textbf{(54)}$$

## ❖ <u>Pinched Plasma:</u>

A large current through an ionised gas creates a large magnetic field, if the current flows in a cylindrical column the lines of force form concentric circles with the cylinder. The lateral repulsion between the lines exerts a pressure radially inwards, the value at any point being by $H^2/8\pi$, where $H$ is the magnetic field intensity. As the charge carriers are free to move in space, the discharge is consequently compressed inwards. This is the Pinch effect. In other word, "Pinch effect is the compression of electrically conducting medium by magnetic field when a huge amount of current flow through this conducting medium".

Now we remember some pervious discussions:

(1) Within discharge tube apply high voltage such that corresponding higher electric field can break the "Debye shielding" and total tube fill-up with plasma. Now due to current flow generate magnetic field. This magnetic field confined the charge particle within circular or helical path.

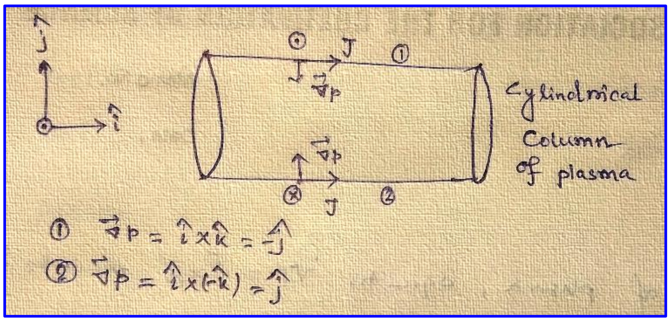
(2) MHD equation: $\rho\left(\frac{d\vec{v}}{dt}\right) = -\vec{\nabla}p + \rho\vec{g} + \vec{J}\times\vec{B}$.

(3) For stable plasma (plasma confinement), the net force $\rho\left(\frac{d\vec{v}}{dt}\right)$ must be zero, just like mechanics. For plasma the gravitational force $\rho\vec{g}$ can be neglected because of very lower mass of plasma gas. Now the MHD equation become,

$$0 = -\vec{\nabla}p + \vec{J}\times\vec{B} \qquad \Rightarrow \vec{\nabla}p = \vec{J}\times\vec{B} \qquad (55)$$

which is the condition for stable plasma (if we confined plasma such that it remains in plasma state for long time within that region).

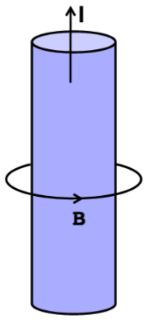
(4) Consider a cylindrical column of plasma, as presented in figure. In all the surface of the cylinder have an inward pressure gradient, which result the plasma as pinched condition.




## An elementary theory of the pinch effect:

The configuration of the pinch effect is illustrated in Figure. A current flowing in the axial direction interacts with a self-produced magnetic field to produce an inwardly directed $\vec{J} \times \vec{B}$ force. If we adopt a magnetohydrodynamic model for the equilibrium case we can write the equation of force equilibrium

$\vec{\nabla} p = \vec{J} \times \vec{B}$ And the Ampere's law states that,




$$\oint \vec{B}.d\vec{l} = \mu_0 \int \vec{J}.d\vec{S} \qquad (56)$$

In the following figure, the equations (55) and (56) changes as,

$$\frac{dp}{dr} = -JB \quad (55a) \quad \text{and} \quad 2\pi r B = \mu_0 I(r) \quad (56a)$$

where $I(r)$ is the current flowing within a radius $r$. It can readily be shown that these two equations lead to the following result, whatever the current distribution $I\ (r)$.

$$2Nk(T_e + T_i) = \frac{\mu_0}{4\pi} I^2 \qquad (57)$$

This result is known as the ***Bennett relation***. $N$ is the number of electrons or ions per unit length of the plasma column, $I$ is the total current, $k$ is Boltzmann's constant and $\mu_0$ is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7} H/m$. [Remember $p = NkT$ and Maxwell's equations for derivation of *Bennett relation*]

## Relation between magnetic field ($\vec{B}$) and plasma pressure (p):

The MHD equation by neglecting the gravitational effect: $\rho\left(\frac{d\vec{v}}{dt}\right) = -\vec{\nabla} p + \vec{J} \times \vec{B}$

$$\Rightarrow \vec{F} = -\vec{\nabla} p + \vec{J} \times \vec{B} \quad \text{or,} \quad \vec{J} \times \vec{B} = \vec{F} + \vec{\nabla} p \qquad (58)$$

The Maxwell's equation, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\Rightarrow \vec{B} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{B} \times \vec{J}$$

$$\Rightarrow \vec{B} \times \vec{J} = \vec{\nabla}\left(\frac{B^2}{2\mu_0}\right) - \frac{(\vec{B}.\vec{\nabla})\vec{B}}{\mu_0}$$ [*Using vector identity,* $\vec{A} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}\left(\frac{A^2}{2}\right) - (\vec{A}.\vec{\nabla})\vec{A}$]

$$\Rightarrow \vec{J} \times \vec{B} = \frac{(\vec{B}.\vec{\nabla})\vec{B}}{\mu_0} - \vec{\nabla}\left(\frac{B^2}{2\mu_0}\right) \qquad (59)$$

Now comparing RHS of the equation of (58) and (59)

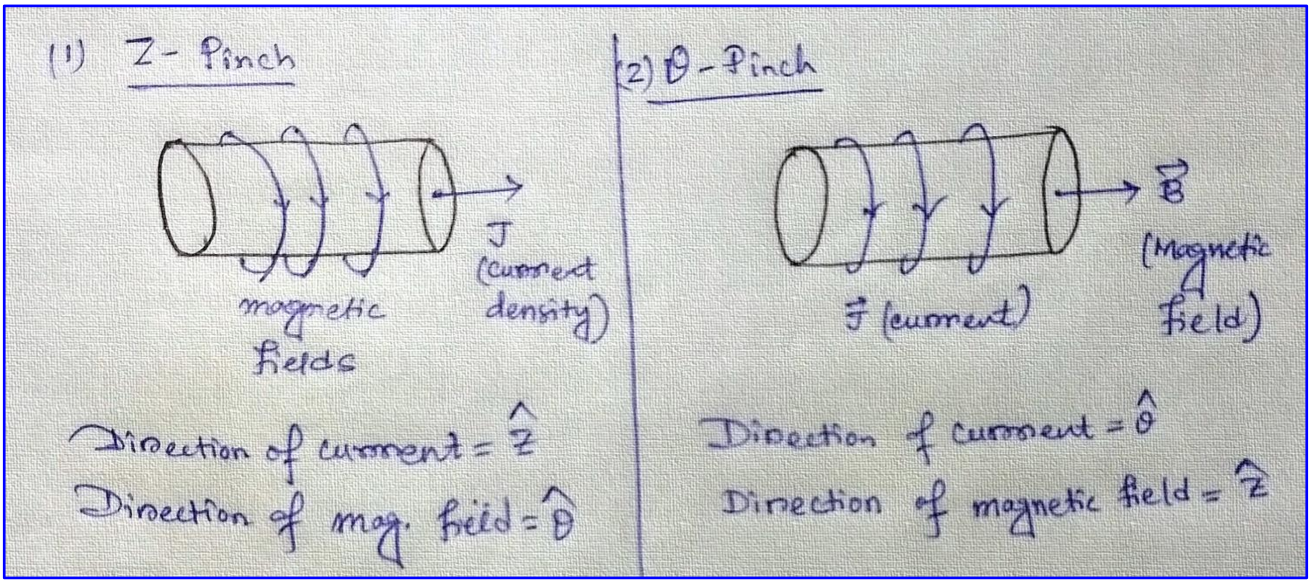
$$\vec{F} = \frac{(\vec{B}.\vec{\nabla})\vec{B}}{\mu_0} \qquad \text{[Relation between } B \text{ and } F] \qquad (60)$$

and $$p = -\frac{B^2}{2\mu_0} \quad \text{Or,} \quad |p| = \frac{B^2}{2\mu_0} \qquad \text{[Relation between } B \text{ and } p] \qquad (61)$$

Due to magnetic field we can create the pressure inside the plasma.

## Type of Pinch effect:

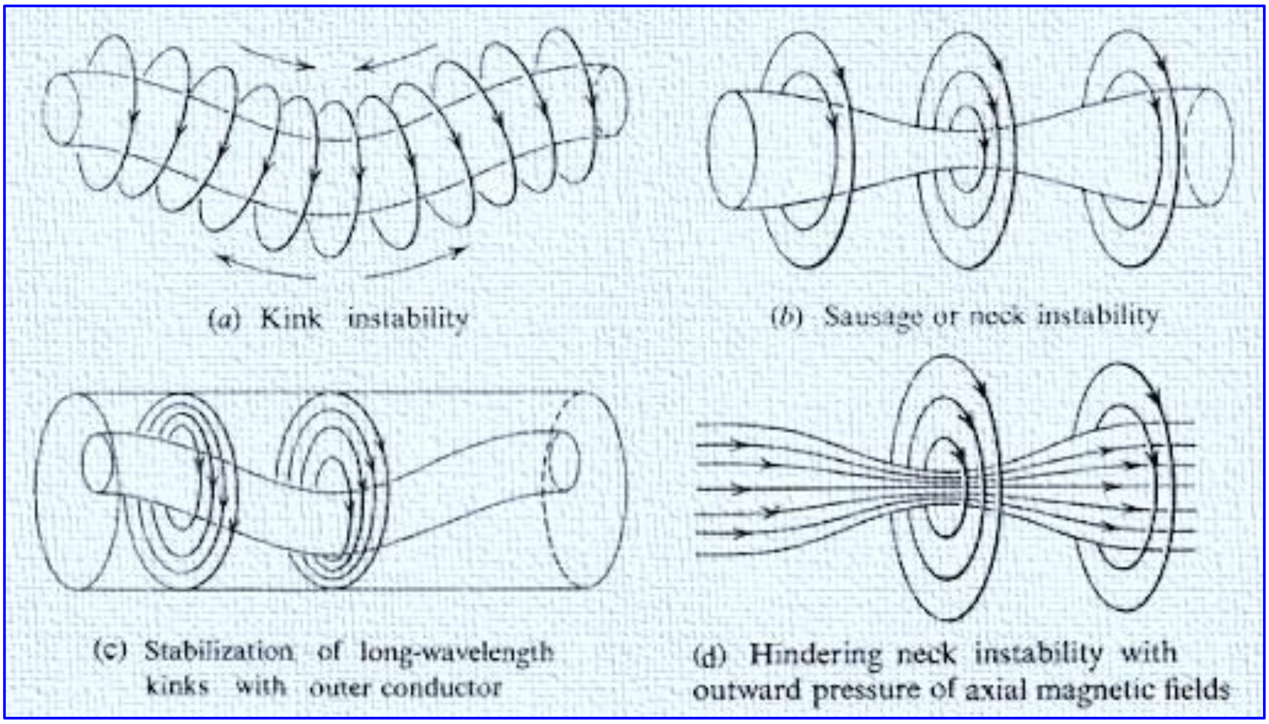
(1) **Z-Pinch** (Zeta Pinch) (2) **Θ-Pinch** (Azimuthal Pinch)




## Instability in Plasma:

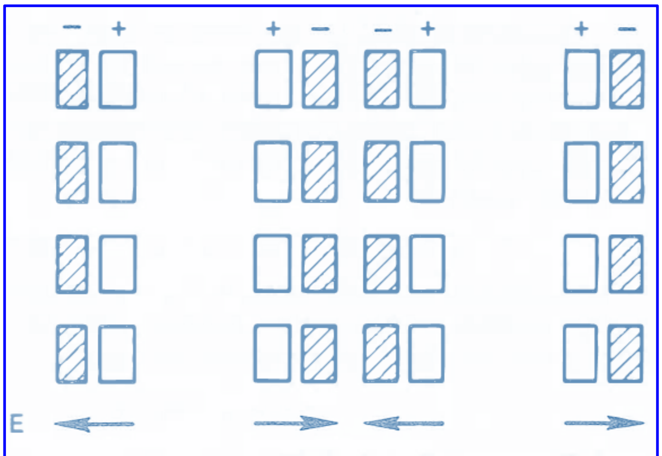
(1) **Kink instability** (Z-Pinch): In case of Z-pinch, the current flow taken in the z-direction of any cylindrical conductor. But in practical the conductor is not uniform, because it is not a rigid conductor, it is plasma column which filed with gases. Since the current not flow in straight path (curve path), the magnetic field are not uniform which result different plasma pressure ($\vec{\nabla} p = \vec{J} \times \vec{B}$) and there is a bending in plasma, which occur instability in plasma.

(2) **Neck or Sausage instability** (θ-Pinch): In case of θ-pinch, the instability also occur, because the radius of current loop is different at different position and result non-uniform magnetic field and inhomogeneous plasma pressure.



## ❖ Plasma Oscillation:

As the electrons in a plasma are displaced from a uniform background of ions, electric fields will be built up in such a direction as to restore the neutrality of the plasma by pulling the electrons back to their original positions. Because of their inertia, the electrons will overshoot and oscillate around their equilibrium positions with a characteristic frequency known as the plasma frequency. This oscillation is so fast that the massive ions do not have time to respond to the oscillating field and may be considered as fixed. In Figure, the open rectangles represent typical elements of the ion fluid, and the darkened rectangles the alternately displaced elements of the electron fluid. The resulting charge bunching causes a spatially periodic $\vec{E}$ field, which tends to restore the electrons to their neutral positions.




Mechanism of plasma oscillations.

To find the plasma frequency $(\omega_p)$ in the simplest case, making the following **assumptions**:

(1) There is ***no magnetic field***;

(2) there are ***no thermal motions*** (KT= 0);

(3) the ***ions are fixed*** in space in a uniform distribution;

(4) the plasma is ***infinite in extent***; and

(5) the electron motions occur only in the $x$ **direction**.

As a consequence of the last assumption, we have

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x}; \qquad \vec{E} = E\hat{x}; \qquad \vec{\nabla}\times\vec{E} = 0; \qquad \vec{E} = -\vec{\nabla}\phi \tag{62}$$

So, no fluctuating magnetic field; this is an electrostatic oscillation.

The electron equations of motion and continuity are:

$$mn_e\left[\frac{\partial\vec{v}_e}{\partial t} + (\vec{v}_e.\vec{\nabla})\vec{v}_e\right] = -en_e\vec{E} \qquad \text{and,} \quad \frac{\partial n_e}{\partial t} + \vec{\nabla}.(n_e\vec{v}_e) = 0. \tag{63}$$

From the Poisson's equation, $$\epsilon_0(\vec{\nabla}.\vec{E}) = \epsilon_0\frac{\partial E}{\partial x} = e(n_i - n_e) \tag{64}$$

The amplitude of oscillation is small, and terms containing higher powers of amplitude factors can be neglected. We first separate the dependent variables into two parts: an "equilibrium" part indicated by a subscript '0', and a "perturbation" part indicated by a subscript '1':

$$n_e = n_0 + n_1; \qquad \vec{v}_e = \vec{v}_0 + \vec{v}_1; \qquad \vec{E} = \vec{E}_0 + \vec{E}_1 \tag{65}$$

The equilibrium quantities express the state of the plasma in the absence of the oscillation. Since we have assumed a uniform neutral plasma at rest before the electrons are displaced, we have

$$\vec{\nabla}n_0 = \vec{v}_0 = \vec{E}_0 = 0; \qquad \text{and,} \qquad \frac{\partial n_0}{\partial t} = \frac{\partial\vec{v}_0}{\partial t} = \frac{\partial\vec{E}_0}{\partial t} = 0. \tag{66}$$

Hence, $$m\left[\frac{\partial\vec{v}_1}{\partial t} + (\vec{v}_1.\vec{\nabla})\vec{v}_1\right] = -e\vec{E}_1. \tag{67}$$

The term $(\vec{v}_1.\vec{\nabla})\vec{v}_1$ is seen to be quadratic in an amplitude quantity, and we shall linearize by neglecting it. The linear theory is valid as long as $|v_1|$ is small enough that such quadratic terms are indeed negligible. Similarly, **equation (63)** becomes,

$$\frac{\partial n_1}{\partial t} + \vec{\nabla}.(n_0\vec{v}_1 + n_1\vec{v}_1) = 0; \Rightarrow \frac{\partial n_1}{\partial t} + \vec{\nabla}.(n_0\vec{v}_1) = \frac{\partial n_1}{\partial t} + n_0\vec{\nabla}.\vec{v}_1 + \vec{v}_1.\vec{\nabla}n_0 = \frac{\partial n_1}{\partial t} + n_0\vec{\nabla}.\vec{v}_1 = 0 \quad (68)$$

We note that, $n_{i0} = n_{e0}$; in equilibrium and that $n_{i1} = 0$; by the assumption of fixed ions, so we have,

$$\epsilon_0(\vec{\nabla}.\vec{E}_1) = -en_1 \quad (69)$$

The oscillating quantities are assumed to behave sinusoidally:

$$\vec{v}_1 = v_1 e^{i(kx-\omega t)}\hat{x}; \qquad n_1 = n_1 e^{i(kx-\omega t)}; \qquad \vec{E} = Ee^{i(kx-\omega t)}\hat{x} \quad (70)$$

The time derivative, $\frac{\partial}{\partial t}$ can therefore be replaced by $-i\omega$, and the gradient, $\vec{\nabla}$ by $ik\hat{x}$. **Equations (67)-(69)** now become;

$$-im\omega v_1 = -eE_1; \qquad -i\omega n_1 = -n_0 ikv_1; \qquad ik\epsilon_0 E_1 = -en_1 \quad (71)$$

Eliminating $n_1$ and $E_1$, $$-im\omega v_1 = -e\frac{-e}{ik\epsilon_0}\frac{-n_0 ikv_1}{-i\omega} = -i\frac{n_0 e^2}{\epsilon_0 \omega v_1} \quad (72)$$

If $v_1$ does not vanish, we must have $$\omega^2 = \frac{n_0 e^2}{m\epsilon_0}$$

The plasma frequency is therefore, $$\omega_p = \left(\frac{n_0 e^2}{m\epsilon_0}\right)^{1/2} \quad (73)$$

Numerically, one can use the approximate formula, $$\omega_p/2\pi = f_p \approx 9\sqrt{n} \quad (74)$$

This frequency, depending only on the plasma density, is one of the fundamental parameters of a plasma. Because of the smallness of mass, the plasma frequency is usually very high. For instance, in a plasma of density $n = 10^{18}\ m^{-3}$, we have $f_p = 9\ GHz$ (Microwave range).

## ❖ Some familiar example of plasma:

(1) **Lighting, Aurora Borealis and electrical sparks**. All these examples show that when an electrical current is passed through plasma, the plasma emits light (electromagnetic radiation).

(2) **Neon and fluorescent light** etc. Electric discharge in plasma provides a rather efficient means of converting electrical energy into light.

(3) **Flame**. The burning gas is weakly ionized. The characteristic yellow colour of a wood flame is produced by 579 nm transition (D lines) of sodium ion.

(4) **Nebula, interstellar gases, the solar wind, the earth's atmosphere, the Van Alen belts**. These provide example of a diffusion, low temperature, ionized gas.

(5) **The sun and the starts**. Controlled thermonuclear fusion in a hot, dense plasma provides us with energy (and entropy!) on earth. Can we develop a practical scheme for trapping this virtually inexhaustible source of energy?

## ❖ Potential applications of plasma:

(1) Plasma may be kept in confinement and heated by magnetic and electric fields. This feature used to build "***plasma gun***" that eject ions at velocity up to 100 km/sec. this gun used in ion rocket engines.

(2) One could build "***plasma motor***", which differ from ordinary motor by having plasma (not metal) as basic conductor of electricity. These motors could be lighter and more efficient than

ordinary motors. Similarly, one can develop "***plasma generator***" to convert mechanical energy is ripe for development. The current "***thermodynamic energy convertors***" are notoriously inefficient in generating electricity.

(3) Plasma could be used as a **resonator or a waveguide**, much like hollow metallic cavities, for electromagnetic radiation. Plasma experiences a whole range of electrostatic and electromagnetic oscillations, which one would be able to put to good use.

(4) **Communication** through the plasma in earth's atmosphere reflects low frequency EM wave (below 1 MHz) and transmit high frequency (above 100 MHz).

| Plasma type | Particle density ($cm^{-3}$) | Temperature (K) | Debye length (cm) | Plasma frequency, $\omega_p$ (Hz) | Collision frequency, $\nu_{en}$ (Hz) |
|---|---|---|---|---|---|
| Interstellar gas | 1 | $10^4$ | $10^3$ | $6 \times 10^4$ | $10^{-4}$ |
| Solar corona | $10^6$ | $10^6$ | 1 | $6 \times 10^7$ | $10^{-1}$ |
| Solar atmospheric gas discharge | $10^{14}$ | $10^4$ | $10^{-4}$ | $6 \times 10^{11}$ | $3 \times 10^9$ |
| Diffuse lab plasma | $10^{12}$ | $10^6$ | $10^{-2}$ | $6 \times 10^{10}$ | $10^5$ |
| Dense lab plasma | $10^{14}$ | $10^6$ | $10^{-4}$ | $6 \times 10^{12}$ | $5 \times 10^8$ |
| Thermonuclear plasma | $10^{16}$ | $10^8$ | $10^{-3}$ | $6 \times 10^{12}$ | $8 \times 10^5$ |

# Assignment-2: Electrodynamics

## (Introductory Plasma Physics) [**LK**]

1. Indicate how sausage instability disrupts a pinched plasma column and indicate how it can be overcome by a longitudinal magnetic field.
2. Explain the principle of magnetic bottle for plasma confinement. What is magnetic mirror?
3. Consider a cylindrical plasma of radius '**R**' with longitudinal current. Prove that the (azimuthal) magnetic field at a distance '**r**'(<**R**) from the centre is given by, $B(r) = \frac{\mu_0}{2\pi r} I(r)$. Where $I(r)$ is the total current flowing through a cylindrical region of radius '**r**'. Hence derive Bennett's relation.
4. Find the field for a highly conducting plasma when the current is confined to the surface.
5. Mansion the principle of two methods for the preparation of laboratory plasma.
6. Write down the magnetohydrodynamic equations describing a dense uncharged plasma.
7. Like what type of fluid does a dense plasma behave?
8. Find the vertical angle for the "loss-cone'. Give an explain of such an arrangement above the earth's atmosphere.
9. Starting from the relevant magnetohydrodynamic equations, derive the equation $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta_m \nabla^2 \vec{B}$ (in usual notation). What are the characteristics of the magnetic field described by the two terms?