Problem Sheet
On
Mechanics

VIVEKANANDA COLLEGE, THAKURPUKUR

NAAC Accredited Grade - A

By

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1. A particle of charge q and mass m moves in a uniform magnetic field $\overrightarrow{B} = B\hat{z}$ and $\overrightarrow{d} = \frac{E}{\rho}\hat{\rho}$ where the constants a and B may be positive or negative. Set up the equations of motion in cylindrical coordinates and show that $m\rho^2\dot{\phi} + \frac{qB}{2c}\rho^2$ is a constant of motion assuming that the force due to electric field is $q\overrightarrow{E}$ and that due to the magnetic field is $\frac{q}{c}\left(\overrightarrow{v} \times \overrightarrow{B}\right)$.

2. Verify if the following forces are conservative or not.

   - $\overrightarrow{F} = \left(\frac{\alpha}{r^4}x,\ \frac{\alpha}{r^4}y,\ \frac{\alpha}{r^4}z\right);$
   - $\overrightarrow{F} = (3z + 5y)\,\hat{i} + (5x + 2z)\,\hat{j} + (2y + 3x + 4z)\,\hat{k},$

   where α is a constant and $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z axes respectively.

3. Prove that the total energy of a particle of mass m acted upon by a central force is given by,
$$E = \frac{h^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$
   where $V(r)$ is the potential energy and h is the angular momentum of the body; $u = \frac{1}{r}$, r and θ being the plane polar coordinates of the particle.

4. Show that the gravitational force is conservative.

5. A particle moves in a force field whose components are given by,
$$\begin{aligned} F_x &= yz\,(1 - 2xyz)\,, \qquad (1)\\ F_y &= zx\,(1 - 2xyz)\,,\\ \text{and}\ \ F_z &= xy\,(1 - 2xyz)\,. \end{aligned}$$
   Verify that the force is conservative and find the potential function from which it is derivable.

6. Two bodies of mass m and M are held at a distance h apart. Only force to be considered is the force of mutual attraction which has a constant value P independent of their separation. Show from momentum and energy consideration that the body of mass m has acquired just before collision a velocity given by $v^2 = \frac{2Ph}{m}\left(1 + \frac{m}{M}\right)$.

7. Show that for a single particle with constant mass, the equation of motion can be put in the form $\frac{dT}{dt} = \overrightarrow{F} \cdot \overrightarrow{v}$, where T is the kinetic energy, $\overrightarrow{F}$ is the force applied and $\overrightarrow{v}$ is the velocity of the particle.

   If the mass varies with time show that the corresponding form is,

$$\frac{d}{dt}(mT) = \overrightarrow{F} \cdot \overrightarrow{p},$$

   where $\overrightarrow{p}$ is the momentum of the particle.

8. A particle moves around a semicircle of radius R from one end A of a diameter to the other end B. It is attracted towards its starting point A by a force proportional to its distance from A. It is also given that when the particle is at B, the force towards A is F_0. Show that the work done against this force when the particle moves around the semi-circle from A to B is $F_0 R$.

9.
   - A particle of mass m moves along the X-axis under the influence of a conservative force field, having a potential $V(x)$. If the particle is located at positions x_1 and x_2 at respective times t_1 and t_2, prove that

$$t_2 - t_1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$$

     where E is the total energy of the particle.
   - If in the above problem $V = \frac{1}{2}kx^2$ and the particle starts from rest at $x = a$, find the equation of motion in terms of x and t.

10. A particle of mass m is at rest at $(a, 0, 0)$ is subjected to a force

$$\overrightarrow{F} = -\left(\frac{\alpha}{x^3}\right)\hat{i},$$

   where α is a constant. Find the time taken by the particle to reach the origin $(a > 0, \alpha > 0)$.