STUDY MATERIAL

VIVEKANANDA COLLEGE THAKURPUKUR

NAAC ACCREDITED – ‘A’

Subject: PHYSICS

Topic: Magnetism [2^{nd} yr. Honours]

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Magnetic vector potential

Magnetic vector potential **A**

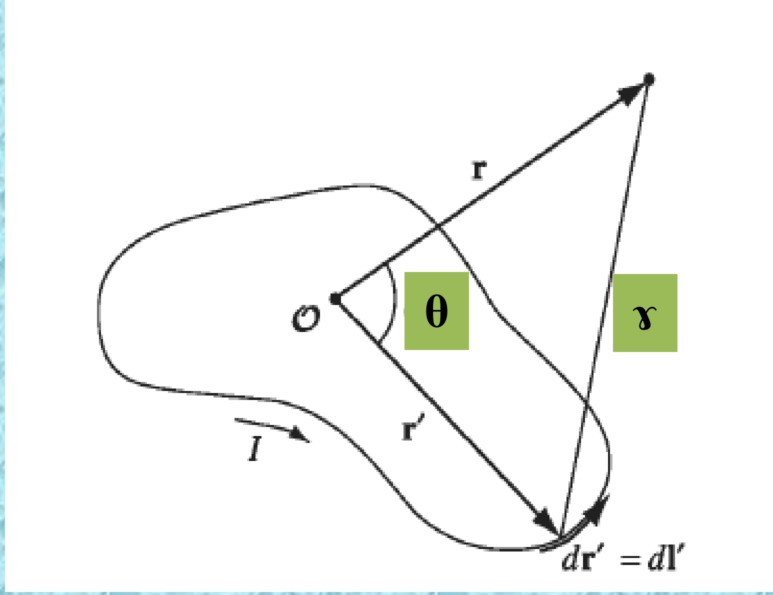
$$\vec{B} = \vec{\nabla} \times \vec{A} \quad and \quad \vec{\nabla}.\vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \; identical\ to$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \; Poisson's\ eqtn.$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{\gamma} d\tau \quad \Rightarrow \quad \vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{\gamma} d\tau$$

Multipole expantion of magnetic vector potential



In the fig. left we set the origin at ‘O’ and eventually

$$\frac{1}{\gamma} = \frac{1}{\sqrt{r^2 + (r')^2 - 2rr'\cos\theta}}$$

$$= \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta)$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{1}{\gamma} dl$$

$$= \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint (\mathrm{r}')^n P_n(cos\theta) dl$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} [\frac{1}{r} \oint dl + \frac{1}{r^2} \oint \mathrm{r}' \cos\theta \mathrm{dl}$$

$$+ \frac{1}{r^3} \oint (\mathrm{r}')^2 \left(\frac{3}{2} cos^2\theta - \frac{1}{2}\right) dl + \cdots$$

The 1st term corresponds to monopole, 2nd to dipole and 3rd to quadrupole

Now as $\oint dl = 0$

Magnetic monopole doesn't exist which reflects In the Maxwell's equation $\vec{\nabla}.\vec{B} = 0$

The Magnetic dipole term

$$\vec{A}_{dip} = \frac{\mu_0 I}{4\pi r^2}\oint \mathrm{r'cos\theta dl}$$

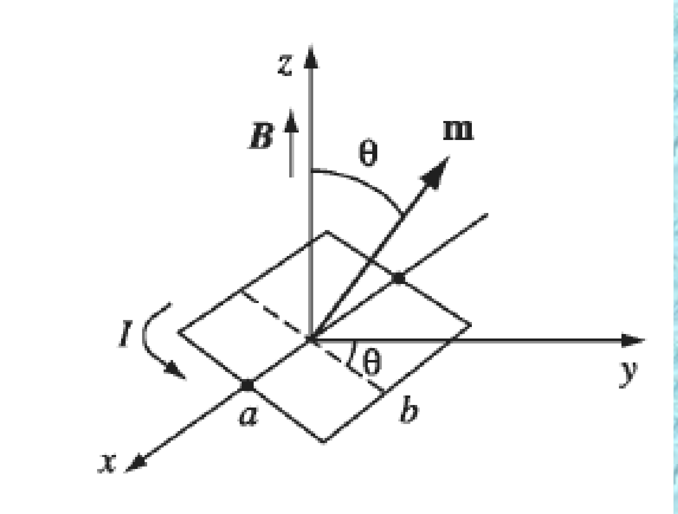
Magnetic dipole moment

$$\vec{A}_{dip} = \frac{\mu_0 I}{4\pi r^2} \oint (\hat{r}.r')dl$$

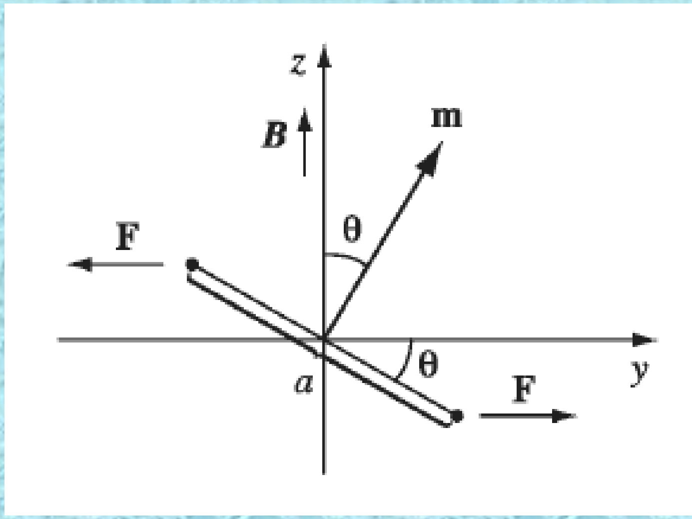
$$\vec{A}_{dip} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Where $\vec{m} \equiv I \int d\vec{a} = I\vec{a}$

is Magnetic dipole moment



Torque on magnetic dipole: $\vec{N} = \vec{m} \times \vec{B}$



Force on magnetic dipole: $\vec{F} = \nabla(\vec{m}.\vec{B})$

Magnetization

Magnetization **M** is defined as magnetic dipole moment per unit volume, so dipole term can be written as

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\overrightarrow{M} \times \hat{r}}{r^2}\, d\tau$$

and as $\nabla \frac{1}{r} = \frac{\hat{r}}{r^2}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int [\vec{M} \times \left(\nabla \frac{1}{r}\right)] d\tau$$

$$= \frac{\mu_0}{4\pi}\int \frac{1}{r}[\vec{\nabla} \times \vec{M}] d\tau + \frac{\mu_0}{4\pi}\oint \frac{1}{r}[\vec{M} \times d\vec{a}]$$

Looks like potential of a volume current

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

Looks like potential of a surface current

$$\vec{K}_b = \vec{M} \times \hat{n}$$

Bound Currents

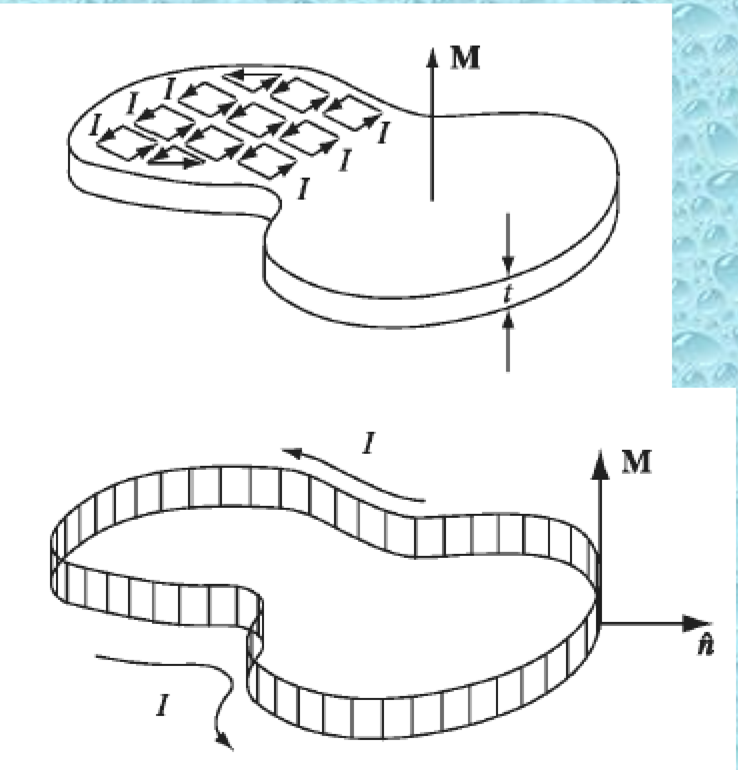
$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}_b}{r}\,d\tau + \frac{\mu_0}{4\pi}\oint \frac{\vec{K}_b}{r}\,da$$

$\mathbf{J_b}$ and $\mathbf{K_b}$ are bound currents analogous to

Bound volume charge $\rho_b = -\vec{\nabla}.\vec{P}$

and bound surface charge $\sigma_b = \vec{P}.\hat{n}$

Bound Currents

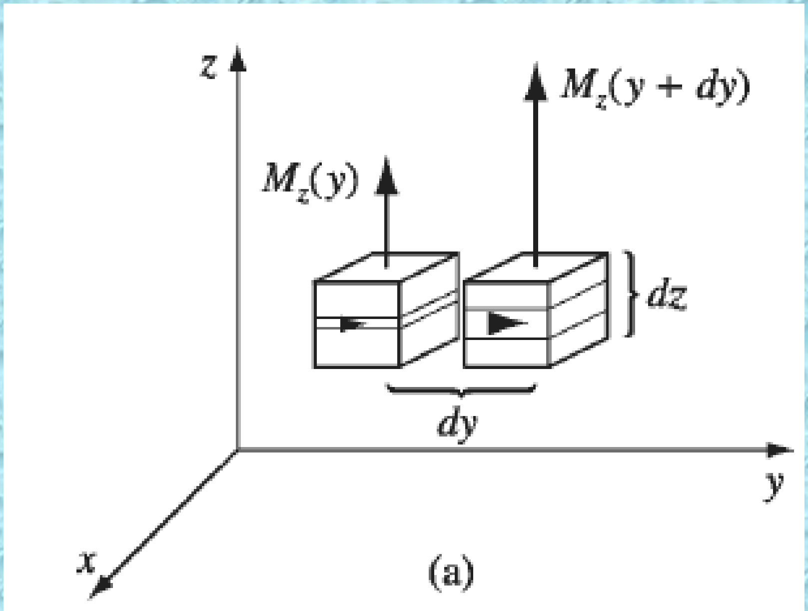


Electrons inside the material generating dipoles by tiny current loops, all the internal currents cancel except at the edge, giving a single current I flowing around the boundary.

Each tiny loop has area a and thickness t, so dipole moment m=Mat. In terms of current m=Ia . So I=Mt and

K_b = I/t = M. Using outward-drawn vector $\vec{K}_b = \vec{M} \times \hat{n}$

Nonuniform Magnetization

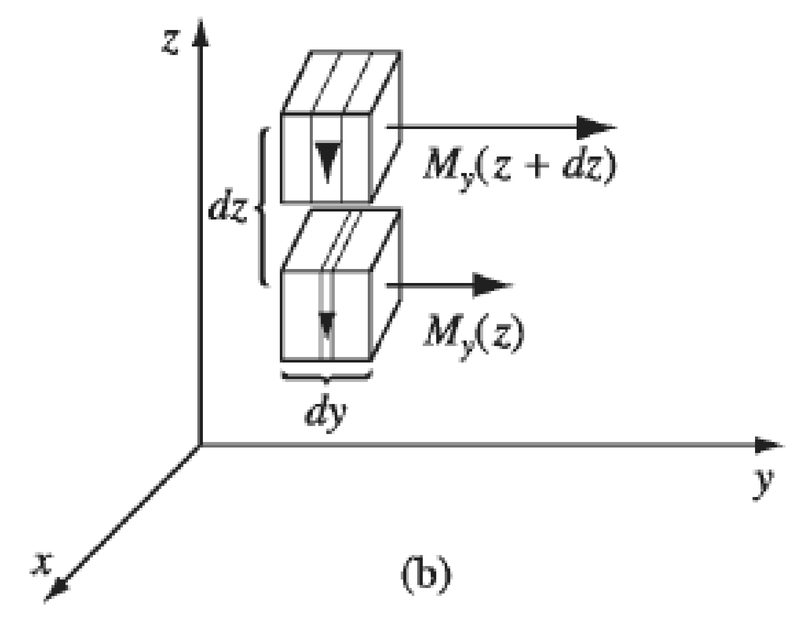


Internal currents no longer cancel. From left the net current in the x dirtn. Is

$$I_x = [M_z(y + dy) - M_z(y)]dz$$
$$= \frac{\partial M_z}{\partial y} dy dz$$

and

$$(J_b)_x = \frac{\partial M_z}{\partial y}$$



Nonuniform magnetization in y dirtn. would contribute

$$-\frac{\partial M_y}{\partial z}$$

So, $(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$

In general $\vec{J_b} = \vec{\nabla} \times \vec{M}$

J_b obeys conservation law

$$\vec{\nabla}.\vec{J_b} = 0$$

Thank You