STUDY MATERIAL

VIVEKANANDA COLLEGE

THAKURPUKUR

NAAC ACCREDITED GRADE—'A'

STUDY MATERIAL

SUBJECT- PHYSICS

TOPIC- DIELECTRIC PORPERTY OF MATTER

NAME OF THE TEACHER:- SUBHAYAN BISWAS

DIELECTRIC PROPERTIES OF SOLID

SOLID STATE PHYSICS 2(25 MARKS)

Polarization. Local Electric Field at an Atom. Depolarization Field. Electric Susceptibility. Polarizability. Clausius Mosotti Equation.

Review of basic formula and definitions:-

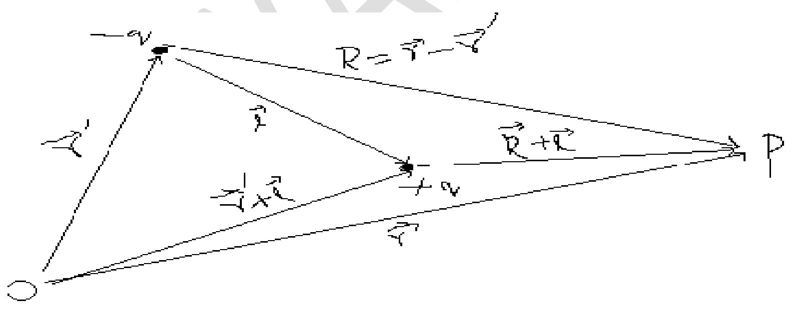
When two very large amount of charge with same value but opposite in nature are separated by a very small distance, then the system of charge is known as electric dipole. The effect of dipole is represented by dipole moment which is defined as the product of value of charge and their distance.

+q ←———————— **-q** the dipole moment **p**= q.**d** in the direction to from –q

←—— **d** ——→ to +q.

For ideal dipole q→∞ and d→0.

ELECTRIC FIELD DUE TO A DIPOLE.



The electric field of a dipole at a point P is

$$\mathrm{E} = \frac{1}{4\pi\varepsilon}\left(\frac{3(\boldsymbol{p}.\boldsymbol{r}).\boldsymbol{r} - r2.\boldsymbol{p}}{r5}\right)$$

Torque on a dipole:-

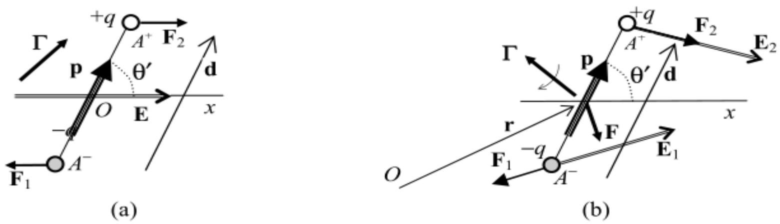


Figure 2.7. *a) Forces exerted by a uniform electric field on a dipole, and b) forces exerted by a non-uniform field on a dipole*

Γ = − pE sin θ = p **× E**. Γ = (−OA) × F – (+OA) × F = q (+OA − (−OA)) × E = q d × E = p ×E.

Consider now the case of a non-uniform electric field and the charges −q and +q located at r + = r − ½ d and r − = r + ½ d /2 (Figure 2.7b).If E varies slowly over the distance d , we may write its components as power series of d up to the first order E α (r ± ½ d) = E α (x ± ½d x , y ± ½d y , z ± ½d z) = E α (x, y, z) ± ½d x (∂ x E α) ± ½d y (∂ y E α) ± ½d z (∂ z E α) = qE α (r) ± ½(d. ∇ ∇)E α .

The resultant of the electric forces acting on the dipole has the components

F α = q[E α (r + d /2) − E α (r − d /2)] = q(d. ∇ ∇) E α (r) = (p. ∇ ∇) E α (r).

Polarization vector: -

Polarization **P** is defined as dipole moment density or net dipole moment in unit volume. If n is number density of dipole of dipole moment **p** then **P=N.p.**

This polarization vector can be defined as

P$=\frac{lim}{\Delta v\rightarrow 0}.\ (\frac{1}{\Delta v})\sum p_i$. or p_i = (P.dv)/N.

Bound charge due to polarization:-

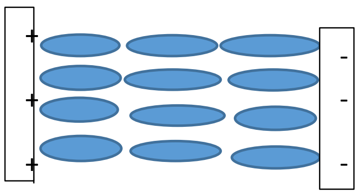
If we place a piece of material inside an electric field, electrons are attracted towards the +ve plate and medium become partially polarized. Due to polarization of any dielectric medium some kind of charge appear in the volume and some on the surface of the medium. These charge are called bound or polarization charge. These charges can't move like free electron of conductor and create an electric field opposite to the applied field inside a dielectric. So net field inside a dielectric medium get diminished by small amount.

Potential due to bound charge

$\varphi(r)=\frac{1}{4\pi\varepsilon_0}\int\frac{\mathbf{p}.\mathbf{n}ds}{R}+\int(\frac{-\nabla.\boldsymbol{P}}{R})dv$. Where P is the polarization vector for uniform polarization charge become zero.

Gauss law in dielectric

$\oint \boldsymbol{E}.\boldsymbol{ds} = \frac{Q_t}{\varepsilon_0} = \frac{Q_b}{\varepsilon_0} + \frac{Q_f}{\varepsilon_0}$. Where $\boldsymbol{Q_b}$ = -$\int \boldsymbol{p}.\boldsymbol{n}ds$(p is opposite to E).



D=$\oint \boldsymbol{E}.\boldsymbol{ds} = \frac{Q_f}{\varepsilon_0}+\frac{1}{\varepsilon_0}(-\int \boldsymbol{p}.\boldsymbol{n}ds)$. $\boldsymbol{Q_f}=\int(\boldsymbol{\varepsilon_0}.\boldsymbol{E}+\boldsymbol{P}).\boldsymbol{ds} = \int \boldsymbol{D}.\boldsymbol{ds}$.

Where **D**=$(\boldsymbol{\varepsilon_0}.\boldsymbol{E}+\boldsymbol{P}) = (\boldsymbol{\varepsilon_0}.\boldsymbol{E}+\boldsymbol{\varepsilon_0}.\boldsymbol{EX}) = \boldsymbol{\varepsilon_0}.\boldsymbol{E}(\boldsymbol{1}+\boldsymbol{X}) = \boldsymbol{\varepsilon_0}.\boldsymbol{\varepsilon_r}.\boldsymbol{E}$

Where$\varepsilon_r = (\boldsymbol{1}+\boldsymbol{X})$. $\boldsymbol{\varepsilon_r}$ = k Is known as dielectric constant.

Dielectric constant, polarizability, and local field:-

Polarization depends on External field as **P** =$\boldsymbol{\varepsilon_0}.\boldsymbol{EX}$ but $\boldsymbol{X}$ depends on material medium.

Now for applying external electric field internal electric field also produced which is smaller than **E**. **p** depends on $\boldsymbol{\varepsilon}$ as **p**=**α.E.** the constant α is called polarizability of a molecule. P is dipole moment of one molecule.

Now **P=N.p**=N**α.E.** [the field must be small such that nonlinear effect can be neglected.]

$$\mathbf{D}=(\boldsymbol{\varepsilon_0}.\boldsymbol{E}+\boldsymbol{P}) = (\boldsymbol{\varepsilon_0}.\boldsymbol{E}+\mathrm{N}\boldsymbol{\alpha}.\mathbf{E}.) = \boldsymbol{\varepsilon_0}(1+\mathrm{N}\boldsymbol{\alpha}/\boldsymbol{\varepsilon_0}).\mathbf{E}.$$

D=$\boldsymbol{\varepsilon_0}.\boldsymbol{E}(\boldsymbol{1}+\boldsymbol{X})$ i.e. $\boldsymbol{X}$= N$\boldsymbol{\alpha}/\boldsymbol{\varepsilon_0}$ or $\boldsymbol{\alpha} = \boldsymbol{X}.\frac{\boldsymbol{\varepsilon_0}}{\boldsymbol{N}}$.

Now N is no. of molecule per unit volume and if m is mass of a molecule then density ρ=N.m=N.M/N_A . [N_A is Avogadro no.]

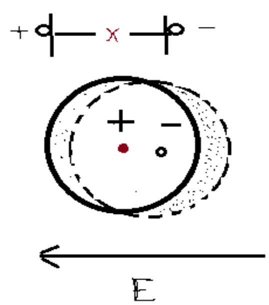
- ⇨ N= (ρ. N_A)/M.
- ⇨ $\boldsymbol{\varepsilon_r} = (\boldsymbol{1}+(\rho.\ N_A\ \boldsymbol{\alpha})/(\boldsymbol{\varepsilon_0}\mathrm{M}.))$

This expression indicate that $\boldsymbol{\varepsilon_r}$ depends on density linearly. So **p**=**α.E.** But it does not hold good for solid and liquid where density variation is small.

DIFFERENT TYPE OF POLARIZABILITY:-

1. Electronic polarizability:-

For a non-polar atom or molecule the effect of all negative charge coincide with total positive charge at one point. i.e. –ve charge center and +ve charge center coincide at a point. Now if the molecule is placed inside an electric field –ve cloud slightly shifted towards opposite to field direction and +ve center shifted in field direction.



In this process polarization occurs and known as electronic polarizability.

This type of polarization is explained by rare gas atom as the atomic interaction is neglected.

Now without external field the –ve charge density surrounding the +ve charge nucleus is $\rho = (3.Z.e/4\pi R^3)$ (where R is radius of the atom).

When external field is applied on the atom it separate the charge centers by an amount x. force on the nucleus

=Z.e*[charge enclosed by the sphere of radius $x/4\pi\boldsymbol{\varepsilon_0}x^2$]

$=Z.e^*(4/3\ \pi x^3\ \rho)/\ 4\pi\boldsymbol{\varepsilon_0}x^2$

$= -\ Z^2e^2x/\ (4\pi\boldsymbol{\varepsilon_0}R^3)$ [Coulombic attraction force between two charge center]

$=-\ Z.e\ \mathbf{E}$ [This force must be equal to Lorentz force.]

$\mathbf{E}= Z.e.x/\ (4\pi\boldsymbol{\varepsilon_0}R^3)$.

Or $\mathbf{x=E.}\ (4\pi\boldsymbol{\varepsilon_0}R^3)/\ Z.e$

Now $\mathbf{p=\alpha.E.=Ze.x}$

So $\underline{\alpha=(4\pi\varepsilon_0 R^3)}$. This is electronic polarizability.

ORIENTATIONAL POLARIZABILITY:-

Langevin's theory of polarization in polar dielectric:-

Some compound as H_2O, HCl have permanent dipole moment. In action of electric field these molecules are oriented in the direction of the applied field and contribute a polarization. The action of electric field are two fold-

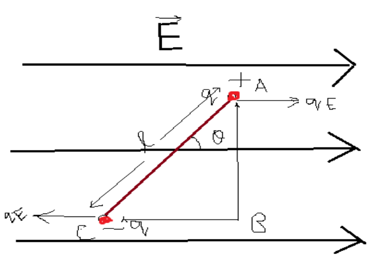
1. It tends to align molecules so that the dipole axes are in the direction of field.

It cause a displacement of electrons in each atom relative to the nucleus of the atom.

Now if there are N no. of molecule are aligned in same direction total polarization **P=N.p**

This polarization is proportional to the applied field and inversely proportional to temperature i.e. due to thermal agitation dipole orient itself in random direction.

The probability of finding a particle with molecular energy W at temp. T is proportional to $[\exp(-W/k_BT)]$.



Let us consider an electric field E is applied in +ve direction and dipole AC is placed in the field with an angle θ.

The torque on the dipole $\tau = p.E \sin\theta$.

The potential energy or work done to rotate the dipole $W = -p.E.\cos\theta$.

Now the no. of molecule (dN) having energy W and inclined in between θ to θ+dθ is

$dN = C.\exp(-W/k_BT)\sin\theta.d\theta$

$\quad = -C.\exp(p.E.\cos\theta/k_BT).d(\cos\theta)$ $\quad [a = (p.E/k_BT).]$

$N = -C\int_0^{\pi/2} e^{a.\cos\theta}\, d(\cos\theta)$

$N = \frac{C}{a}(e^a - e^{-a})$

$C = \frac{Na}{(e^a - e^{-a})}$.

Now dipole moment in the direction of applied field before orientation is $dM = dN.p.\cos\theta$

$M = \int dM = \int dN.p.\cos\theta = p.\int_0^{\pi} e^{a\cos\theta} \cos\theta \sin\theta.d\theta$

$= -pC.\int_1^{-1} e^x.x.dx \qquad [x = \cos\theta]$

$$M=p.C\left[\frac{e^{a}}{a}+\frac{e^{-a}}{a}-\frac{1}{a^{2}}(e^{a}-e^{-a})\right.$$

$$M=p.\frac{Na}{(e^{a}-e^{-a})}.\left[\frac{e^{a}}{a}+\frac{e^{-a}}{a}-\frac{1}{a^{2}}(e^{a}-e^{-a})\right]$$

$$M=p.N.\left[\frac{e^{a}+e^{-a}}{e^{a}-e^{-a}}-\frac{1}{a}\right]$$

M=p.N. [coth (a)-1/a]

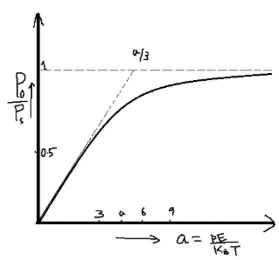
This dipole moment per unit volume is called polarization.

P_o= p.N.[coth(a)-1/a]=pN.L(a) where L(a) is Langevin's function.

pN=dipolemoment per unit volume & P_o become maximum =pN for all dipole aligned in the direction of field =P_s

P_o= P_s L (a) = P_s. [coth (a)-1/a]

When a<<1 L(a)≈a/3, i.e. (P_o/P_s)=a/3

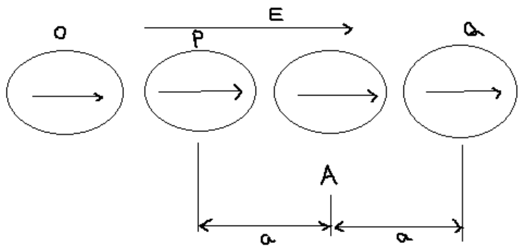


P_o=p.N. (p.E /3.k_BT) = α_0NE [where α_0 = (p^2 /3.k_BT) is known as orienatiolal polarizability.]

INTERNAL OR LOCAL FIELD IN LIQUID AND SOLID:-

In solid and liquid the atoms are so closely packed that the only external field is not responsible but influenced by neighboring dipoles. So one atom sees the total effect of external and internal field. This total field is known as local field. But this is not a macroscopic effect.

It can be represented by a simple one dimensional array of dipole.



The atom 'A' have dipole direction parallel to 'E' field and its dipole moment must be influenced by 'P','Q' and 'O' etc atom.

The radial and transverse component of electric field due to the dipole is $\mathbf{E_r} = (1/4\pi\boldsymbol{\varepsilon_0})\,(2p\,cos\theta/r^3)\,\mathbf{ar}$ & $\mathbf{E_\theta} = (1/4\pi\boldsymbol{\varepsilon_0})\,(p\,sin\theta/r^3)\mathbf{a_\theta}$

Then for 'P' atom r=a, θ=0. And for 'Q' atom r=a θ=180^0

⇨ $\mathbf{E_{AP}} = (1/4\pi\boldsymbol{\varepsilon_0})(2p/a^3)$; for 'Q' atom $\mathbf{E_{AQ}} = (1/4\pi\boldsymbol{\varepsilon_0})(-2p/a^3)$

⇨ **So** total field due to 'P' and 'Q' $E = (1/4\pi\boldsymbol{\varepsilon_0})\{(2p/a^3) - (-2p/a^3)\}$

⇨ $\mathbf{E} = p/\pi\boldsymbol{\varepsilon_0}\, a^3$

Similarly for 'O' and 'R' $E = p/\pi\boldsymbol{\varepsilon_0}\,(2a)^3$ and so on.

Total local field $E_i(A) = E + p/\pi\boldsymbol{\varepsilon_0}\, a^3 + p/\pi\boldsymbol{\varepsilon_0}\,(2a)^3 + \ldots\ldots\ldots.$

$E_i\,(A) = E + p/\pi\boldsymbol{\varepsilon_0}\, a^3(1 + 1/2^3 + 1/3^3 + \ldots\ldots)$

$E_i\,(A) = E + (p/\pi\boldsymbol{\varepsilon_0}\, a^3)*1.2 \qquad \sum_1^\infty \frac{1}{n^3} \approx 1.2$

$E_i\,(A) = E + (1.2*p/\pi\boldsymbol{\varepsilon_0}\, a^3)$

In 3 dimensional case this field may be very complicated and depends on crystal structure. But for simplicity we can replace ($1/a^3$) by N (molecular no. density) and ($1.2/\pi$) by a constant γ which depends on the structure ($\sum_1^\infty \frac{1}{n^3} \approx 1.2$).

So for 3 D array local field $E_i = E + \gamma p.N/\varepsilon_0$ {p=q.l, N= no. of atom in unit volume}

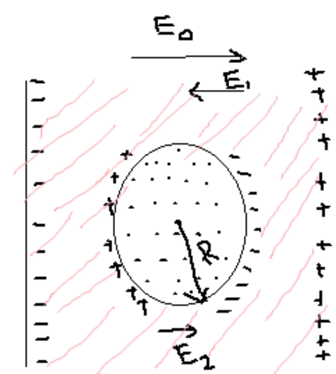
$\mathbf{E_i = E + P\ (\gamma/\varepsilon_0)}$

$P = N.E_l.\alpha = \varepsilon_0 \chi E = \varepsilon_0 (\varepsilon_r - 1) E$

$(\varepsilon_r - 1) = (N.E_l.\alpha / \varepsilon_0 E)$.

EVALUATION OF THE LOCAL FIELD FOR CUBIC STRUCTURE:-

To evaluate E_L we must calculate the total field acting on a certain typical dipole. this field being due to the external field as well as all other dipoles in the system. The dipole is imagined to be surrounded by a spherical cavity of radius R is sufficiently large that the matrix outside of it may be treated as a continuous medium as far as the dipole is concerned. The interaction of our dipole with the other dipoles lying inside the cavity.



The local field acting on the central dipole, is thus given by the sum---

$\mathbf{E_L = E_o + E_p + E_s + E_m}$

Where $\mathbf{E_o}$ is the external field.

$\mathbf{E_p}$ is the field due to polarization charges lying on the internal surface of the plate.

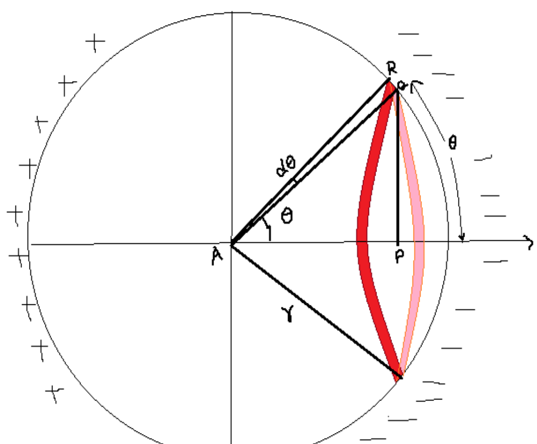
$\mathbf{E_s}$ field due to induced charge on the surface of the Lorentz sphere due to polarization

$\mathbf{E_m}$ is field due to other dipole inside the sphere.

$\mathbf{E_o}=\mathbf{D}/\varepsilon_0=\mathbf{E}+\mathbf{P}/\ \varepsilon_0$

$E_p=-P/\ \varepsilon_0$

Further if we consider a crystal of high symmetry (cubic crystal) $E_m=0$ as the dipole inside the sphere is randomly oriented.



Lorentz sphere

Calculation of E_s:-

Enlarge view of the cavity is shown in the figure. If dA is the surface area of the strip on the sphere of radius r lying between θ to θ+dθ where θ is the angle with the direction of reference of applied field, then $dA = 2\pi.\ (P.Q)(Q.R) = 2\pi\ r\ \sin\theta.\ (r.d\theta) = 2\pi\ r^2\ \sin\theta.d\theta$

Now dq is the total charge on the surface dA and N is the total dipole in area dA.

$\mathbf{P}=(N/V)p = (N\ dq.d/dA.d) = N.dq/dA$

$N.dq = \mathbf{P.dA}=P.dA.\cos\theta = P.\ 2\pi\ r^2\ \sin\theta.d\theta.\ \mathrm{Cos}\theta$

$\mathbf{E_s} = \int_0^{\pi} \frac{N.dq.cos\theta}{4\pi\varepsilon_0 r^2} = \int_0^{\pi} \frac{\mathrm{P.2\pi\ r2\ sin\theta.d\theta.cos\theta}.cos\theta}{4\pi\varepsilon_0 r^2} = \frac{P}{2\varepsilon_0}\int_0^{\pi} \frac{\mathrm{sin\theta.d\theta}.cos^2\theta}{r^2}$

$\mathbf{E_s} = -\frac{\mathbf{P}}{2\varepsilon_0}\int_0^{\pi} \frac{\mathrm{d(cos\theta)}.cos^2\theta}{r^2} = -\frac{\mathbf{P}}{2\varepsilon_0}.(cos^3\theta)/3 = \frac{\mathbf{P}}{3\varepsilon_0}$

So total field $\mathbf{E_L = E_o + E_p + E_s + E_m}$

$\mathbf{E_L} = (\mathbf{E}+\mathrm{P}/\varepsilon_0) + (-\mathrm{P}/\varepsilon_0) + \frac{\mathbf{P}}{3\varepsilon_0} + 0 = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$

RELATION BETWEEN RELATIVE PERMITIVITY or CLAUSIUS MOSOTTI RELATION:-

Dipolemoment of a single atom of solid or liquid is proportional to local field i.e. $\mathbf{p} = \mathbf{E_L}.\alpha$

$\mathbf{P} = \mathrm{N}.\mathbf{p} = \mathrm{N}\ \mathbf{E_L}.\alpha$

Now we know $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$

$\mathbf{P} = \varepsilon\mathbf{E} - \varepsilon_0\mathbf{E} = \varepsilon_r.\varepsilon_0\mathbf{E} - \varepsilon_0\mathbf{E} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$

$\mathrm{N}\ \mathbf{E_L}.\alpha = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$

$(\varepsilon_r - 1) = \mathrm{N}\ \mathbf{E_L}.\alpha/\varepsilon_0\mathbf{E}$

We know the relation between local field and applied field as

$\mathbf{E_L} = (\mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}) = (\mathbf{E} + \frac{\mathrm{N}\ E_L\alpha}{3\varepsilon_0})$

$\mathbf{E} = \mathbf{E_L}(1 - \mathrm{N}\alpha/3\varepsilon_0)$

$\mathbf{E_L/E = 1/}$ $(1\text{-}N\alpha/3\varepsilon_0) = 3\varepsilon_0/(3\varepsilon_0\text{-}N\alpha)$

$(\varepsilon_r - 1) = 3N\alpha/(3\varepsilon_0\text{-}N\alpha)$

$(\varepsilon_r - 1 + 3) = (\varepsilon_r + 2) = 3N\alpha/(3\varepsilon_0\text{-}N\alpha) + 3 = 9\varepsilon_0/(3\varepsilon_0\text{-}N\alpha)$

$(\varepsilon_r - 1)/(\varepsilon_r + 2) = N\alpha/3\varepsilon_0$

This relation known as Clausius Mosotti relation.

Multiplying both side by molar volume M/ρ

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} * \frac{M}{\rho} = \frac{NM\alpha}{3 * \rho * \varepsilon_0} = \frac{N_A\alpha}{3 * \rho * \varepsilon_0}$$

From this equation 'α' can be calculated very easily if we know molar mass and density and relative permittivity.