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Quantification Theory

10

10.1 The Need for Quantification
10.2 Singular Propositions
10.3 Universal and Existential Quantifiers
10.4 Traditional Subject–Predicate Propositions
10.5 Proving Validity
10.6 Proving Invalidity
10.7 Asyllogistic Inference

10.1 The Need for Quantification

Many valid deductive arguments cannot be tested using the logical techniques of the preceding two chapters. Therefore we must now enhance our analytical tools. We do this with *quantification*, a twentieth century development chiefly credited to Gottlob Frege (1848–1945), a great German logician and the founder of modern logic. His discovery of quantification has been called the deepest single technical advance ever made in logic.

To understand how quantification increases the power of logical analysis, we must recognize the limitations of the methods we have developed so far. The preceding chapters have shown that we can test deductive arguments effectively—but only arguments of one certain type, those whose validity depends entirely on the ways in which simple statements are truth-functionally combined into compound statements. Applying elementary argument forms and the rule of replacement, we draw inferences that permit us to discriminate valid from invalid arguments of that type. This we have done extensively.

When we confront arguments built of propositions that are *not* compound, however, those techniques are not adequate; they cannot *reach* the critical elements in the reasoning process. Consider, for example, the ancient argument

All humans are mortal.

Socrates is human.

Therefore Socrates is mortal.*

*It was to arguments of this type that the classical or Aristotelian logic was primarily devoted, as described in Chapters 5 and 6. Those traditional methods, however, do not possess the generality or power of the newer symbolic logic and cannot be extended to cover all deductive arguments of the kinds we are likely to confront.

This argument is obviously valid. But using the methods so far, introduced we can only symbolize it as

A

H

$\therefore M$

and on this analysis it appears to be invalid. What is wrong here? The difficulty arises from the fact that the validity of this argument, which is intuitively clear, depends on the *inner logical structure* of its premises, and that inner structure cannot be revealed by the system of symbolizing statements that we have developed thus far. The symbolization immediately above, plainly too blunt, is the best we can do without quantifiers. That is because the propositions in this valid argument are not compound, and the techniques presented thus far, which are designed to deal with compound statements, cannot deal adequately with noncompound statements. A method is needed with which noncompound statements can be described and symbolized in such a way that their inner logical structure will be revealed. The theory of **quantification** provides that method.

Quantification enables us to interpret noncompound premises as compound statements, without loss of meaning. With that interpretation we can then use all the elementary argument forms and the rule of replacement (as we have done with compound statements), drawing inferences and proving validity or invalidity—after which the compound conclusion reached may be transformed (again using quantification) back into the noncompound form with which we began. This technique adds very greatly to the power of our analytical machinery.

The methods of deduction developed earlier remain fundamental; quantifiers do not alter the rules of inference in any way. What has gone before may be called the *logic of propositions*. We now proceed, using some additional symbolization, to apply these rules of inference more widely, in what is called the *logic of predicates*. The inner structure of propositions, the relations of subjects and predicates, is brought to the surface and made accessible by *quantifiers*. Introducing this symbolization is the next essential step.

10.2 Singular Propositions

We begin with the simplest kind of noncompound statement, illustrated by the second premise of the illustrative argument above, "Socrates is human." Statements of this kind have traditionally been called *singular propositions*. An

affirmative singular proposition asserts that a particular individual has some specified attribute. *Socrates* is the subject term in the present example (as ordinary grammar and traditional logic both agree), and *human* is the predicate term. The subject term denotes a particular individual; the predicate term designates some attribute that individual is said to have.

The same subject term, obviously, can occur in different singular propositions. We may assert that "Socrates is mortal," or "Socrates is fat," or "Socrates is wise," or "Socrates is beautiful." Of these assertions, some are true (the first and the third), and some are false (the second and fourth.)* Similarly, the very same predicate term can occur in different singular propositions. The term *human* is a predicate that appears in each of the following: "Aristotle is human," "Brazil is human," "Chicago is human," and "O'Keeffe is human"—of which the first and fourth are true, while the second and third are false.

An "individual" in this symbolism can refer not only to persons, but to any individual *thing*, such as a country, a book, a city, or anything of which an *attribute* (such as human or heavy) can be predicated. Attributes do not have to be adjectives (such as "mortal" or "wise") as in our examples thus far, but can also be nouns (such as "a human"). In grammar the distinction between adjective and noun is important, of course, but in this context it is not significant. We do not need to distinguish between "Socrates is mortal" and "Socrates is a mortal." Predicates can also be verbs, as in "Aristotle writes," which can be expressed alternatively as "Aristotle is a writer." The critical first step is to distinguish between the subject and the predicate terms, between the individuals and the attributes they may be said to have. We next introduce two different kinds of symbols for referring to *individuals* and to *attributes*.

To denote individuals we use (following a very widely adopted convention) small, or lowercase letters, from *a* through *w*. These symbols are **individual constants.** In any particular context in which they may occur, each will designate one particular individual throughout the whole of that context. It is usually convenient to denote an individual by the first letter of its (or his or her) name. We may use the letter *s* to denote Socrates, *a* to denote Aristotle, *b* to denote Brazil, *c* to denote Chicago, and so forth.

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*Here we follow the custom of ignoring the time factor, and use the verb "is" in the tenseless sense of "is, will be, or has been." Where considerations of time change are crucial, the somewhat more complicated symbolism of the logic of relations is required for an adequate treatment.

We use capital letters to symbolize attributes that individuals may have, and again it is convenient to use the first letter of the attribute referred to: *H* for human, *M* for mortal, *F* for fat, *W* for wise, and so forth.

We can now symbolize a singular proposition. By writing an attribute symbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified. Thus the singular proposition, "Socrates is human," will be symbolized simply as *Hs*. And, of course, *Ha* symbolizes "Aristotle is human," *Hb* symbolizes "Brazil is human," *Hc* symbolizes "Chicago is human," and so forth.

It is important to note the pattern that is common to these terms. Each begins with the same attribute symbol, *H*, and is followed by a symbol for some individual, *s* or *a* or *b* or *c*, and so forth. We could write the pattern as "*H*—" where the dash to the right of the predicate symbol is a place marker for some individual symbol. This pattern we symbolize as *Hx*. We use *Hx* [sometimes written as *H*(*x*)] to symbolize the common pattern of all singular propositions that attribute "being human" to some individual. The letter *x* is called an **individual variable**—it is simply a place marker, indicating where the various individual letters *a* through *w* (the individual constants) may be written. When one of those constants does appear in place of *x*, we have a singular proposition. The letter *x* is available to serve as the variable because, by convention, *a* through *w* are the only letters we allow to serve as individual constants.

Let us examine the symbol *Hx* more closely. It is called a *propositional function*. We define a **propositional function** as an expression that (1) contains an individual variable and (2) becomes a statement when an individual constant is substituted for the individual variable.* So a propositional function is not itself a proposition, although it can become one by substitution. Individual constants may be thought of as the proper names of individuals. Any singular proposition is a substitution instance of a propositional function; it is the result of substituting some individual constant for the individual variable in that propositional function.

A propositional function normally has some true substitution instances and some false substitution instances. If *H* symbolizes human, *s* symbolizes Socrates, and *c* symbolizes Chicago, then *Hs* is true and *Hc* is false. With the substitution made, what confronts us is a proposition; before the substitution is made, we have only the propositional *function*. There are an unlimited number of such propositional functions, of course: *Hx*, and *Mx*, and *Bx*, and *Fx*,

*Some writers regard "propositional functions" as the meanings of such expressions, but here we define them to be the expressions themselves.

and *Wx*, and so on. We call these propositional functions *simple predicates*, to distinguish them from more complex propositional functions to be introduced in following sections. A **simple predicate** is a propositional function that has some true and some false substitution instances, each of which is an affirmative singular proposition.

10.3 Universal and Existential Quantifiers

A singular proposition affirms that some individual thing has a given predicate, so it is the substitution instance of some propositional function. If the predicate is *M* for mortal, or *B* for beautiful, we have the simple predicates *Mx* or *Bx*, which assert humanity or beauty of nothing in particular. If we substitute Socrates for the variable *x*, we get singular propositions, "Socrates is mortal," or "Socrates is beautiful." But we might wish to assert that the attribute in question is possessed by more than a single individual. We might wish to say that "Everything is mortal," or that "Something is beautiful." These expressions contain predicate terms, but they are not singular propositions because they do not refer specifically to any particular individuals. These are *general* propositions.

Let us look closely at the first of these general propositions, "Everything is mortal." It may be expressed in various ways that are logically equivalent. We could express it by saying "All things are mortal." Or we could express it by saying:

Given any individual thing whatever, it is mortal.

In this latter formulation the word "it" is a relative pronoun that refers back to the word "thing" that precedes it. We can use the letter *x*, our individual variable, in place of both the pronoun and its antecedent. So we can rewrite the first general proposition as

Given any x, x is mortal.

Or, using the notation for predicates we introduced in the preceding section, we may write

Given any x, *Mx*.

We know that *Mx* is a propositional function, not a proposition. But here, in this last formulation, we have an expression that *contains Mx*, and that clearly *is* a proposition. The phrase "Given any *x*" is customarily symbolized by "(*x*)," which is called the **universal quantifier.** That first general proposition may now be completely symbolized as

(x) *Mx*

which says, with great penetration, "Everything is mortal."

"(x)" ↓ For all x

This analysis shows that we can convert a propositional function into a proposition not only by *substitution*, but also by *generalization*, or *quantification*.

Consider now the second general proposition we had entertained: "Something is beautiful." This may also be expressed as

There is at least one thing that is beautiful.

In this latter formulation, the word "that" is a relative pronoun referring back to the word "thing." Using our individual variable *x* once again in place of both the pronoun "that" and its antecedent "thing," we may rewrite the second general proposition as

There is at least one *x* such that *x* is beautiful.

Or, using the notation for predicates, we may write

There is at least one *x* such that *Bx*.

Once again we see that, although *Bx* is a propositional function and not a proposition, we have here an expression that contains *Bx* that *is* a proposition. The phrase "there is at least one *x* such that" is customarily symbolized by "$(\exists x)$" which is called the **existential quantifier.** Thus the second general proposition may be completely symbolized as

$(\exists x)\ Bx$

which says, with great penetration, "Something is beautiful."

Thus we see that propositions may be formed from propositional functions either by **instantiation,** that is, by substituting an individual constant for its individual variable, or by **generalization,** that is, by placing a universal or existential quantifier before it.

Now consider: The *universal* quantification of a propositional function, $(x)Mx$, is true if and only if *all* its substitution instances are true; that is what universality means here. It is also clear that the *existential* quantification of a propositional function, $(\exists x)Mx$, is true if and only if it has *at least one* true substitution instance. Let us assume (what no one would deny) that there exists at least one individual. Under this very weak assumption, every propositional function must have at least one substitution instance, an instance that may or may not be true. But it is certain that, under this assumption, if the *universal* quantification of a propositional function is true, then the *existential* quantification of it must also be true. That is, if every *x* is *M*, then, if there exists at least one thing, that thing is *M*.

Up to this point, only affirmative singular propositions have been given as substitution instances of propositional functions. *Mx* (*x* is mortal) is a propositional function. *Ms* is an instance of it, an affirmative singular proposition that says "Socrates is mortal." But not all propositions are affirmative. One may deny that Socrates is mortal, saying ~*Ms*, "Socrates is not mortal." If *Ms* is a substitution instance of *Mx*, then ~*Ms* may be regarded as a substitution instance of the propositional function ~*Mx*. And thus we may enlarge our conception of propositional functions, beyond the simple predicates introduced in the preceding section, to permit them to contain the negation symbol, "~."

With the negation symbol at our disposal, we may now enrich our understanding of quantification as follows. We begin with the general proposition

> Nothing is perfect.

which we can paraphrase as

> Everything is imperfect.

which in turn may be written as

> Given any individual thing whatever, it is not perfect.

which can be rewritten as

> Given any x, x is not perfect.

If *P* symbolizes the attribute of being perfect, we can use the notation just developed (the quantifier and the negation sign) to express this proposition ("Nothing is perfect.") as $(x) \sim Px$.

Now we are in a position to list and illustrate a series of important connections between universal and existential quantification.

First, the (universal) general proposition "Everything is mortal" is *denied* by the (existential) general proposition "Something is not mortal." Using symbols, we may say that $(x)Mx$ is denied by $(\exists x) \sim Mx$. Because each of these is the denial of the other, we may certainly say (prefacing the one with a negation symbol) that the biconditional

$$\sim(x)Mx \stackrel{T}{\equiv} (\exists x) \sim Mx$$

is necessarily, logically true.

Second, "Everything is mortal" expresses exactly what is expressed by "There is nothing that is not mortal"—which may be formulated as another biconditional, also logically true:

$$\sim(x)Mx \stackrel{T}{\equiv} \sim(\exists x) \sim Mx$$

Third, it is clear that the (universal) general proposition, "Nothing is mortal," is *denied* by the (existential) general proposition, "Something is mortal." In symbols we say that $(x) \sim Mx$ is denied by $(\exists x)Mx$. And because each of these is the denial of the other, we may certainly say (again prefacing the one with a negation symbol) that the biconditional

$$\sim(x)\sim Mx \stackrel{T}{\equiv} (\exists x)Mx$$

is necessarily, logically true.

And fourth, "Everything is not mortal" expresses exactly what is expressed by "There is nothing that is mortal"—which may be formulated as a logically true biconditional:

$$(x)\sim Mx \stackrel{T}{\equiv} \sim(\exists x)Mx$$

These four logically true biconditionals set forth the interrelations of universal and existential quantifiers. We may replace any proposition in which the quantifier is prefaced by a negation sign (using these logically true biconditionals) with another logically equivalent proposition in which the quantifier is not prefaced by a negation sign. We list these four biconditionals again, now replacing the illustrative predicate *M* (for mortal) with the symbol Φ (the Greek letter *phi*), which will stand for *any* simple predicate whatsoever.

$$[(x)\Phi x] \stackrel{T}{\equiv} [\sim(\exists x)\sim\Phi x]$$

$$[(\exists x)\Phi x] \stackrel{T}{\equiv} [\sim(x)\sim\Phi x]$$

$$[(x)\sim\Phi x] \stackrel{T}{\equiv} [\sim(\exists x)\Phi x]$$

$$[(\exists x)\sim\Phi x] \stackrel{T}{\equiv} [\sim(x)\Phi x]$$

Graphically, the general connections between universal and existential quantification can be described in terms of the square array shown in Figure 10-1.

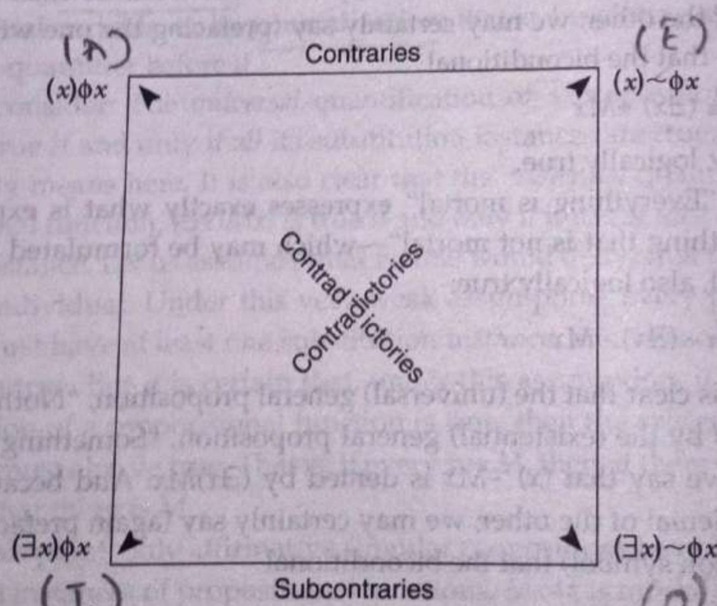

Figure 10-1

Continuing to assume the existence of at least one individual, we can say, referring to this square, that:

1. The two top propositions are *contraries;* that is, they may both be false but they cannot both be true.
2. The two bottom propositions are *subcontraries;* that is, they may both be true but they cannot both be false.
3. Propositions that are at opposite ends of the diagonals are *contradictories,* of which one must be true and the other must be false.
4. On each side of the square, the truth of the lower proposition is implied by the truth of the proposition directly above it.

10.4 Traditional Subject–Predicate Propositions

Using the existential and universal quantifiers, and with an understanding of the square of opposition in Figure 10-1, we are now in a position to analyze (and to use accurately in reasoning) the four types of general propositions that have been traditionally emphasized in the study of logic. The standard illustrations of these four types are the following:

All humans are mortal.	(universal affirmative: A)
No humans are mortal.	(universal negative: E)
Some humans are mortal.	(particular affirmative: I)
Some humans are not mortal.	(particular negative: O)

Each of these types is commonly referred to by its letter: the two affirmative propositions, **A** and **I** (from the Latin *affirmo*, I affirm); and the two negative propositions, **E** and **O** (from the Latin *nego*, I deny).*

In symbolizing these propositions by means of quantifiers, we are led to a further enlargement of our conception of a propositional function. Turning first to the **A** proposition, "All humans are mortal," we proceed by means of successive paraphrasings, beginning with

Given any individual thing whatever, if it is human then it is mortal.

The two instances of the relative pronoun "it" clearly refer back to their common antecedent, the word "thing." As in the early part of the preceding

*An account of the traditional analysis of these four types of propositions was presented in Chapter 5.

section, because those three words have the same (indefinite) reference, they can be replaced by the letter x and the proposition rewritten as

Given any x, if x is human then x is mortal.

Now using our previously introduced notation for "if–then," we can rewrite the preceding as

Given any x, x is human $\supset$ x is mortal.

Finally, using our now-familiar notation for propositional functions and quantifiers, the original **A** proposition is expressed as

$(x)(Hx \supset Mx)$

In our symbolic translation, the **A** proposition appears as the universal quantification of a new kind of propositional function. The expression $Hx \supset Mx$ is a propositional function that has as its substitution instances neither affirmative nor negative singular propositions, but conditional statements whose antecedents and consequents are singular propositions that have the same subject term. Among the substitution instances of the propositional function $Hx \supset Mx$ are the conditional statements $Ha \supset Ma$, $Hb \supset Mb$, $Hc \supset Mc$, $Hd \supset Md$, and so on.

There are also propositional functions whose substitution instances are conjunctions of singular propositions that have the same subject terms. Thus the conjunctions $Ha \bullet Ma$, $Hb \bullet Mb$, $Hc \bullet Mc$, $Hd \bullet Md$, and so on, are substitution instances of the propositional function $Hx \bullet Mx$. There are also propositional functions such as $Wx \vee Bx$, whose substitution instances are disjunctions such as $Wa \vee Ba$ and $Wb \vee Bb$. In fact, any truth-functionally compound statement whose simple component statements are singular propositions that all have the same subject term may be regarded as a substitution instance of a propositional function containing some or all of the various truth-functional connectives: dot, wedge, horseshoe, three-bar equivalence, and curl, in addition to the simple predicates Ax, Bx, Cx, Dx, ... In our translation of the **A** proposition as $(x)(Hx \supset Mx)$, the parentheses serve as punctuation marks. They indicate that the universal quantifier (x) "applies to" or "has within its scope" the entire (complex) propositional function $Hx \supset Mx$.

Before going on to discuss the other traditional forms of categorical propositions, it should be observed that our symbolic formula $(x)(Hx \supset Mx)$ translates not only the standard-form proposition, "All H's are M's," but any other English sentence that has the same meaning. When, for example, a character in Henrik Ibsen's play, *Love's Comedy, says,* "A friend married is a friend lost," that is just another way of saying, "All friends who marry are friends who are lost." There are many ways, in English, of saying the same thing.

Here is a list, not exhaustive, of different ways in which we commonly express universal affirmative propositions in English:

H's are *M*'s.
An *H* is an *M*.
Every *H* is *M*.
Each *H* is *M*.
Any *H* is *M*.
No *H*'s are not *M*.
Everything that is *H* is *M*.
Anything that is *H* is *M*.
If anything is *H*, it is *M*.
If something is *H*, it is *M*.
Whatever is *H* is *M*.
H's are all *M*'s.
Only *M*'s are *H*'s.
None but *M*'s are *H*'s.
Nothing is an *H* unless it is an *M*.
Nothing is an *H* but not an *M*.

To evaluate an argument we must understand the language in which the propositions of that argument are expressed. Some English idioms are a little misleading, using a temporal term when no reference to time is intended. Thus the proposition, "*H*'s are always *M*'s," is ordinarily understood to mean simply that *all H*'s are *M*'s. Again, the same meaning may be expressed by the use of abstract nouns: "Humanity implies (or entails) mortality" is correctly symbolized as an **A** proposition. That the language of symbolic logic has a single expression for the common meaning of a considerable number of English sentences may be regarded as an advantage of symbolic logic over English for cognitive or informative purposes—although admittedly a disadvantage from the point of view of rhetorical power or poetic expressiveness.

Quantification of the A Proposition

The **A** proposition, "All humans are mortal," asserts that if anything is a human, then it is mortal. In other words, for any given thing *x*, if *x* is a human, *then x* is mortal. Substituting the horseshoe symbol for "if–then," we get

Given any x, x is human ⊃ x is mortal.

In the notation for propositional functions and quantifiers this becomes

$(x)\ [Hx \supset Mx]$

Quantification of the E Proposition

The **E** proposition, "No humans are mortals," asserts that if anything is human, then it is not mortal. In other words, for any given thing *x*, *if* *x* is a human, *then* *x* is not mortal. Substituting the horseshoe symbol for "if–then," we get:

Given any x, x is a human ⊃ x is not mortal.

In the notation for propositional functions and quantifiers, this becomes

$(x)\ [Hx \supset {\sim}Mx]$

This symbolic translation expresses not only the traditional **E** form in English, but also such diverse ways of saying the same thing as "There are no *H*'s that are *M*," "Nothing is both an *H* and an *M*," and "*H*'s are never *M*."

Quantification of the I Proposition

The **I** proposition, "Some humans are mortal," asserts that there is at least one thing that is a human *and* is mortal. In other words, there is at least one *x* such that *x* is a human *and* *x* is mortal. Substituting the dot symbol for conjunction, we get

There is at least one x such that x is a human • x is mortal.

In the notation for propositional functions and quantifiers, this becomes

$(\exists x)\ [Hx \cdot Mx]$

Quantification of the O Proposition

The **O** proposition, "Some humans are not mortal," asserts that there is at least one thing that is a human and is not mortal. In other words, there is at least one *x* such that *x* is human *and* *x* is not mortal. Substituting the dot symbol for conjunction we get

There is a least one x such that x is a human • x is not mortal.

In the notation for propositional functions and quantifiers, this becomes

$(\exists x)\ [Hx \cdot {\sim}Mx]$

Where the Greek letters *phi* (Φ) and *psi* (Ψ) are used to represent any predicates whatever, the four general subject–predicate propositions of traditional logic may be represented in a square array as shown in Figure 10-2.

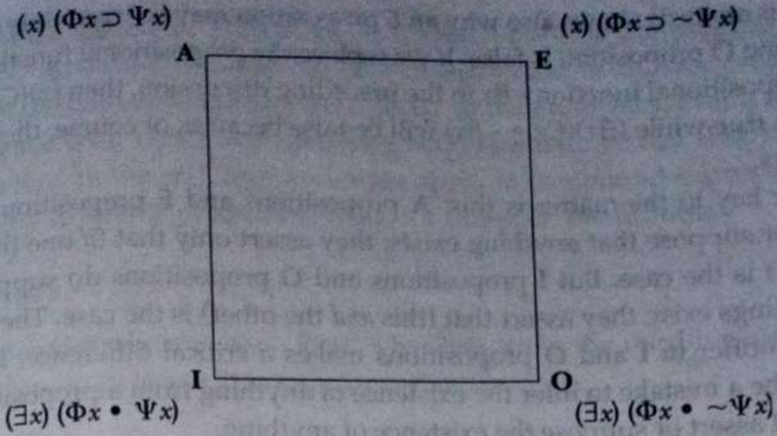

Figure 10-2

Of these four, the **A** and the **O** are contradictories, each being the denial of the other; **E** and **I** are also contradictories.

It might be thought that an **I** proposition follows from its corresponding **A** proposition, and an **O** from its corresponding **E**, but this is not so. An **A** proposition may very well be true while its corresponding **I** proposition is false. Where Φx is a propositional function that has no true substitution instances, then no matter what kinds of substitution instances the propositional function Ψx may have, the universal quantification of the (complex) propositional function $\Phi x \supset \Psi x$ will be true. For example, consider the propositional function, "*x* is a centaur," which we abbreviate as *Cx*. Because there are no centaurs, every substitution instance of *Cx* is false, that is, *Ca*, *Cb*, *Cc*, . . . , are all false. Hence every substitution instance of the complex propositional function $Cx \supset Bx$ will be a conditional statement whose antecedent is false. The substitution instances $Ca \supset Ba$, $Cb \supset Bb$, $Cc \supset Bc$, . . . , are therefore all true, because any conditional statement asserting a material implication must be true if its antecedent is false. Because all its substitution instances are true, the universal quantification of the propositional function $Cx \supset Bx$, which is the **A** proposition $(x)(Cx \supset Bx)$, is true. But the corresponding **I** proposition $(\exists x)(Cx \bullet Bx)$ is false, because the propositional function $Cx \bullet Bx$ has no true substitution instances. That $Cx \bullet Bx$ has no true substitution instances follows from the fact that *Cx* has no true substitution instances. The various substitution instances of $Cx \bullet Bx$ are $Ca \bullet Ba$, $Cb \bullet Bb$, $Cc \bullet Bc$, . . . , each of which is a conjunction whose first conjunct is false, because *Ca*, *Cb*, *Cc*, . . . , are all false. Because all its substitution instances are false, the existential quantification of the propositional function $Cx \bullet Bx$, which is the **I** proposition $(\exists x)(Cx \bullet Bx)$, is false. Hence an **A** proposition may be true while its corresponding **I** proposition is false.

This analysis shows also why an **E** proposition may be true while its corresponding **O** proposition is false. If we replace the propositional function Bx by the propositional function $\sim Bx$ in the preceding discussion, then $(x)(Cx \supset \sim Bx)$ may be true while $(\exists x)(Cx \bullet \sim Bx)$ will be false because, of course, there are no centaurs.

The key to the matter is this: **A** propositions and **E** propositions do not assert or suppose that anything exists; they assert only that (*if* one thing *then* another) is the case. But **I** propositions and **O** propositions do suppose that some things exist; they assert that (this *and* the other) is the case. The existential quantifier in **I** and **O** propositions makes a critical difference. It would plainly be a mistake to infer the existence of anything from a proposition that does not assert or suppose the existence of anything.

If we make the general assumption that there exists at least one individual, then $(x)(Cx \supset Bx)$ does imply $(\exists x)(Cx \supset Bx)$. But the latter is not an **I** proposition. The **I** proposition, "Some centaurs are beautiful," is symbolized as $(\exists x)(Cx \bullet Bx)$, which says that there is at least one centaur that is beautiful. But what is symbolized as $(\exists x)(Cx \supset Bx)$ can be rendered in English as "There is at least one thing such that, if it is a centaur, then it is beautiful." It does not say that there is a centaur, but only that there is an individual that is either not a centaur or is beautiful. This proposition would be false in only two possible cases: first, if there were no individuals at all; and second, if all individuals were centaurs and none of them was beautiful. We rule out the first case by making the explicit (and obviously true) assumption that there is at least one individual in the universe. And the second case is so extremely implausible that any proposition of the form $(\exists x)(\Phi x \supset \Psi x)$ is bound to be quite trivial, in contrast to the significant **I** form $(\exists x)(\Phi x \bullet \Psi x)$. The foregoing should make clear that, although in English the **A** and **I** propositions "All humans are mortal" and "Some humans are mortal" differ only in their initial words, "all" and "some," their difference in meaning is not confined to the matter of universal versus existential quantification, but goes deeper than that. The propositional functions quantified to yield **A** and **I** propositions are not just differently quantified; they are different propositional functions, one containing "$\supset$," the other "$\bullet$." In other words, **A** and **I** propositions are not as much alike as they appear in English. Their differences are brought out very clearly in the notation of propositional functions and *quantifiers*.

For purposes of logical manipulation we can work best with formulas in which the negation sign, if one appears at all, applies only to simple predicates. So we will want to replace formulas in ways that have this result. This we can do quite readily. We know from the rule of replacement established in Chapter 9 that we are always entitled to replace an expression by another that is logically equivalent to it; and we have at our disposal four logical equivalences (listed in Section 10.3) in which each of the propositions in which the quantifier is negated is shown equivalent to another proposition in which the negation sign applies directly to the predicates. Using the rules of inference with which we have long been familiar, we can shift negation signs so that, in the end, they no longer apply to compound expressions but apply only to simple predicates. Thus, for example, the formula

$$\sim(\exists x)(Fx \bullet \sim Gx)$$

can be successively rewritten. First, when we apply the third logical equivalence given on page 444, it is transformed into

$$(x) \sim(Fx \bullet \sim Gx)$$

Then when we apply De Morgan's theorem, it becomes

$$(x)(\sim Fx \vee \sim\sim Gx)$$

Next, the principle of Double Negation gives us

$$(x)(\sim Fx \vee Gx)$$

And finally, when we invoke the definition of Material Implication, the original formula is rewritten as the **A** proposition

$$(x)(Fx \supset Gx)$$

We call a formula in which negation signs apply only to simple predicates a **normal-form formula.**

Before turning to the topic of inferences involving noncompound statements, the reader should acquire some practice in translating noncompound statements from English into logical symbolism. The English language has so many irregular and idiomatic constructions that there can be no simple rules for translating an English sentence into logical notation. What is required in each case is that the meaning of the sentence be understood and then restated in terms of propositional functions and quantifiers.

EXERCISES

A. Translate each of the following into the logical notation of propositional functions and quantifiers, in each case using the abbreviations suggested and making each formula begin with a quantifier, not with a negation symbol.

EXAMPLE

1. No beast is without some touch of pity. (*Bx*: *x* is a beast; *Px*: *x* has some touch of pity.)